From basic electrical concepts to equivalent circuits that models physical phenomena

Enrique Hernández Balaguera
Department of Applied Mathematics, Materials Science and Engineering and Electronic Technology
Universidad Rey Juan Carlos, Madrid (Spain)

15-19 May 2023, Chania, Creta (Greece)
Erasmus+ Staff Training Mobility

Universidad Rey Juan Carlos

HELLENIC MEDITERRANEAN UNIVERSITY

Erasmus+

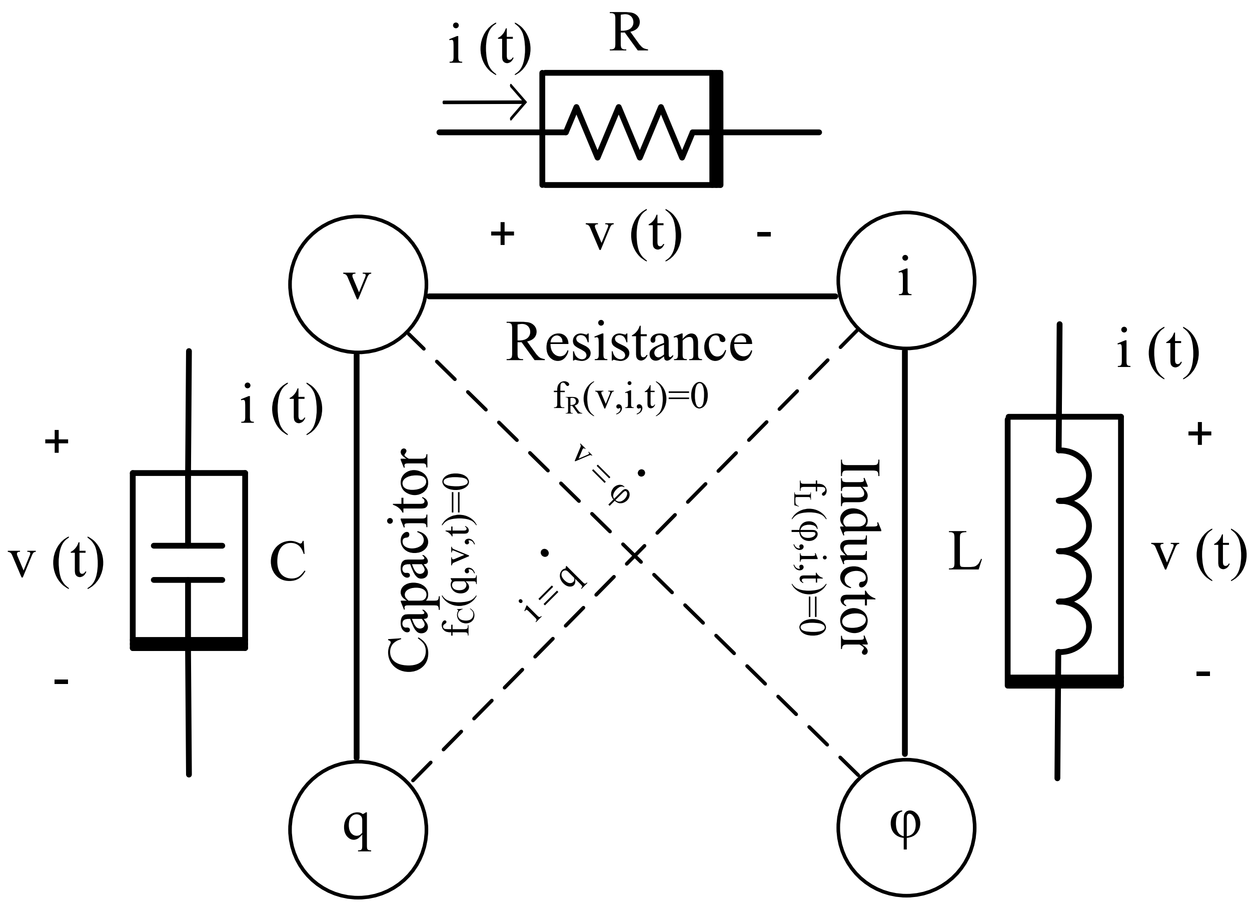
10th International Week
2nd ATHENA International Week

INTRODUCTION

Basic electrical elements in circuit theory

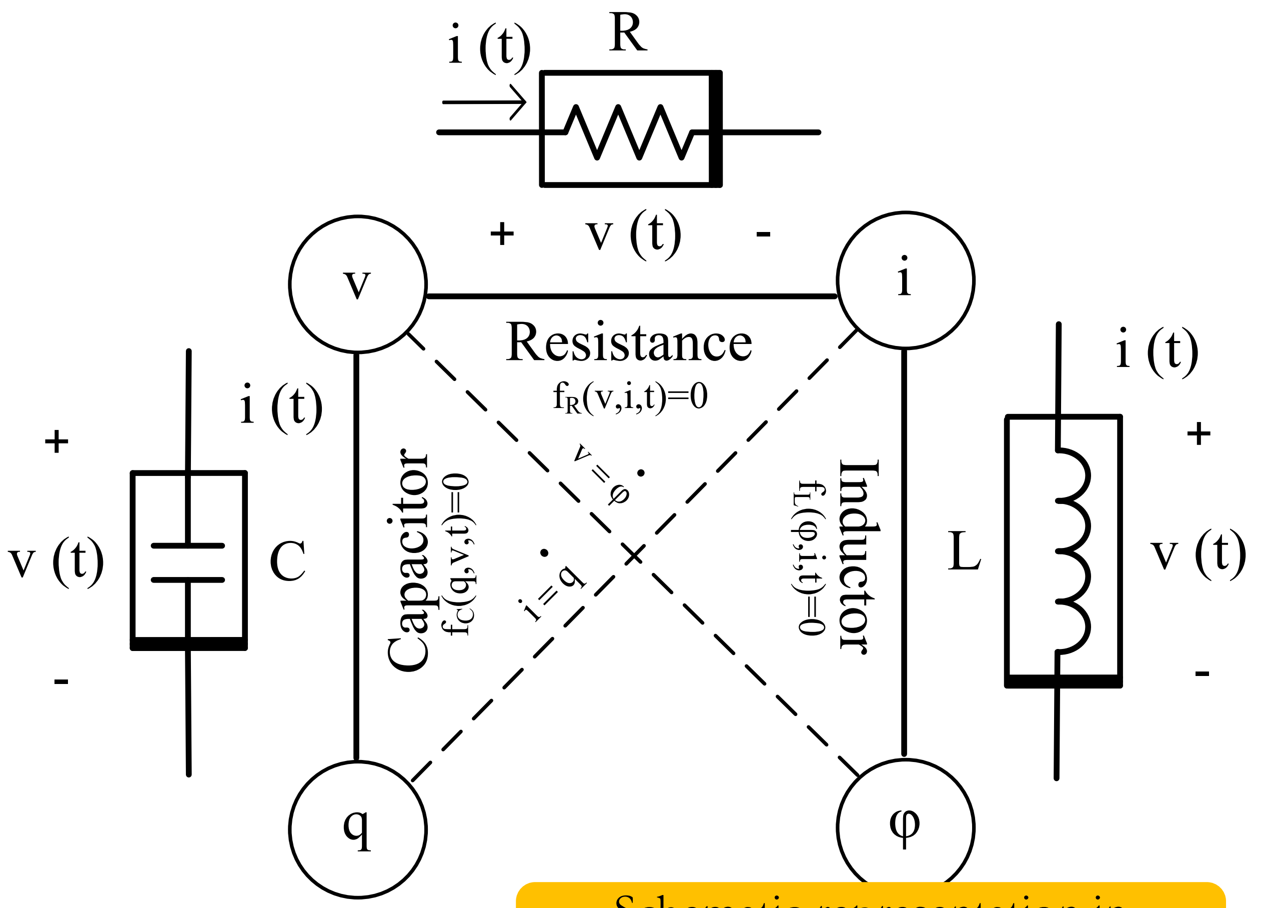
INTRODUCTION

Basic electrical elements in circuit theory



INTRODUCTION

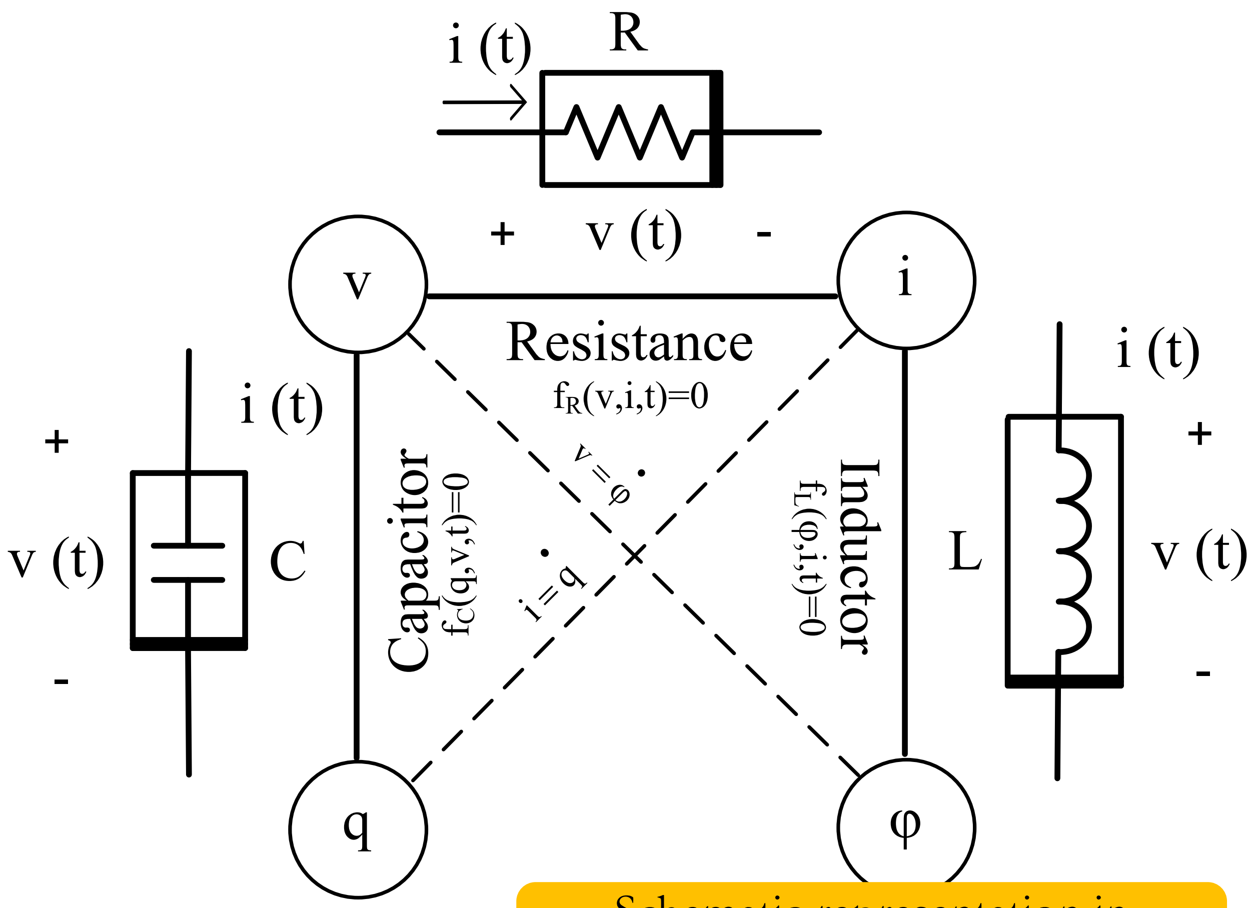
Basic electrical elements in circuit theory



Schematic representation in network analysis and synthesis

INTRODUCTION

Basic electrical elements in circuit theory

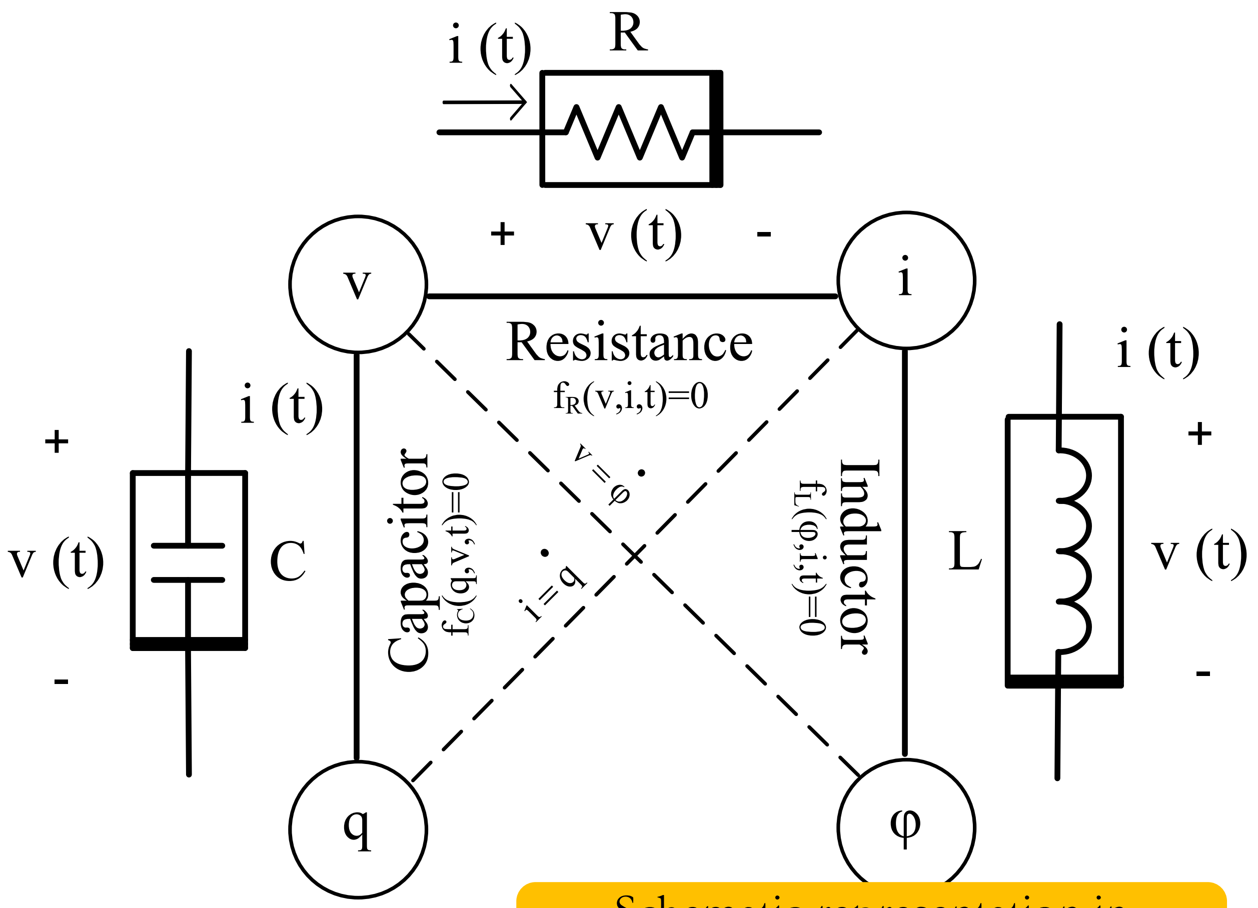


Schematic representation in network analysis and synthesis

- **Components:** Resistance, capacitor, and inductor.

INTRODUCTION

Basic electrical elements in circuit theory

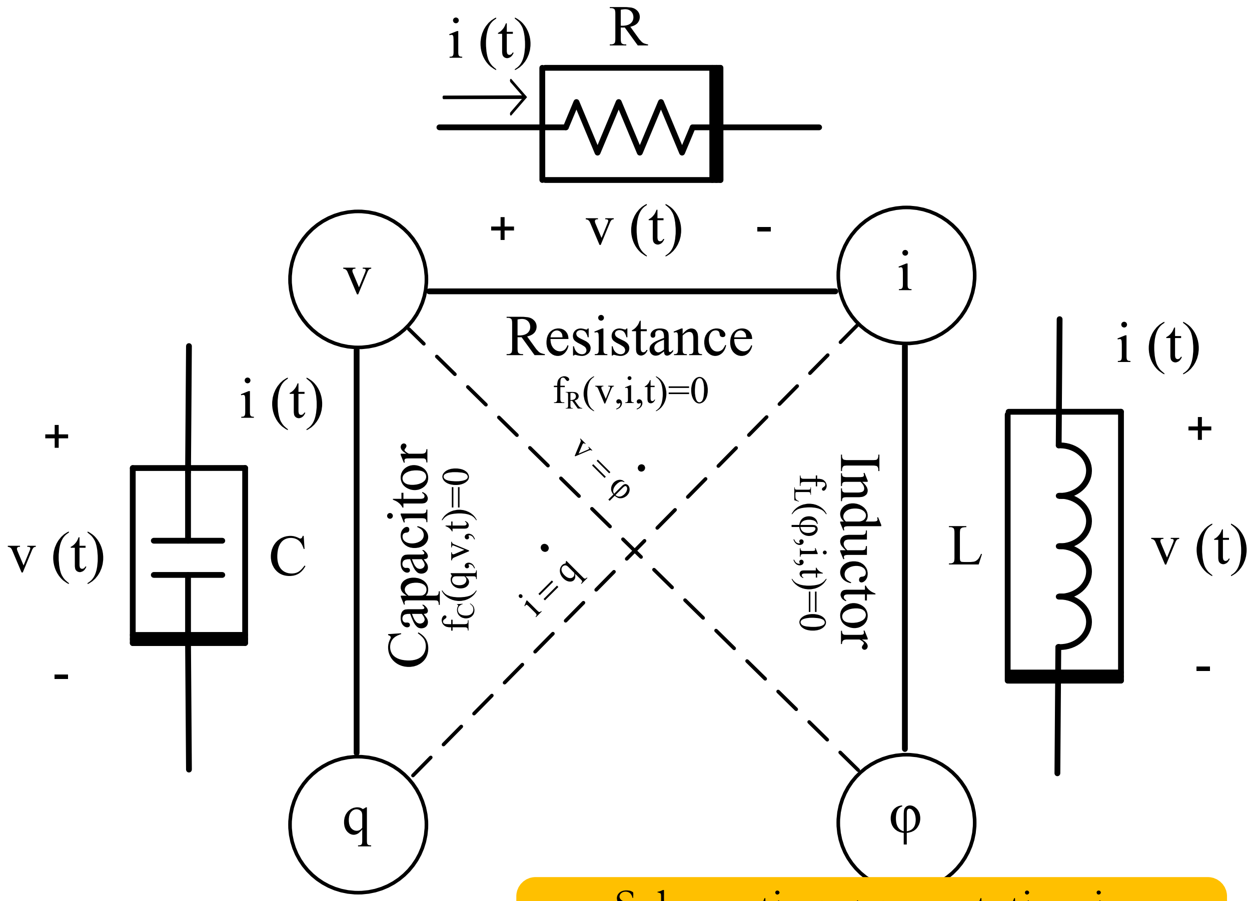


Schematic representation in network analysis and synthesis

- **Components:** Resistance, capacitor, and inductor.
- **Interest variables:** voltage $v(t)$, current $i(t)$, el magnetic flux $\varphi(t)$, and electrical charge $q(t)$.

INTRODUCTION

Basic electrical elements in circuit theory



Schematic representation in network analysis and synthesis

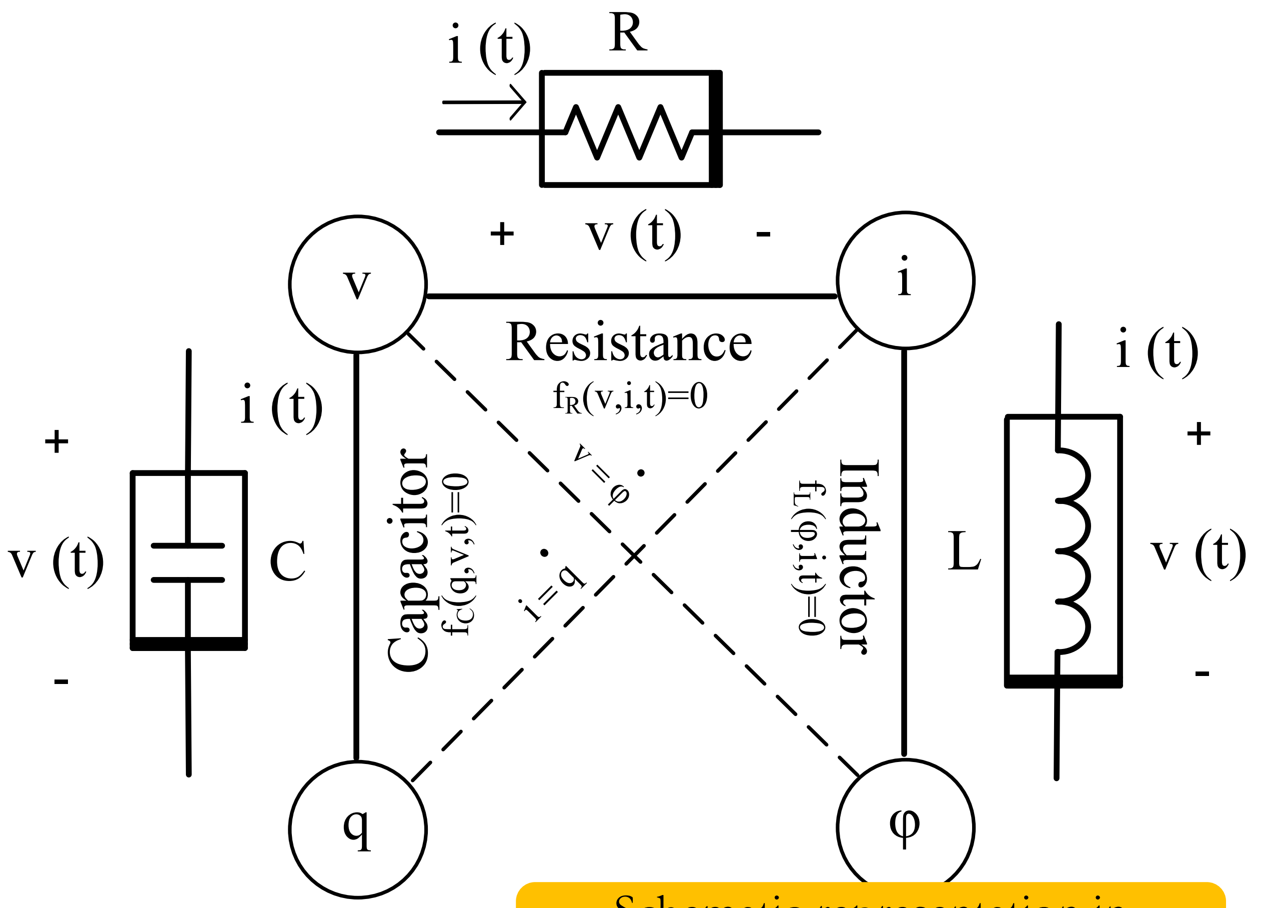
- **Components:** Resistance, capacitor, and inductor.
- **Interest variables:** voltage $v(t)$, current $i(t)$, el magnetic flux φ(t), and electrical charge $q(t)$.
- **Relationships:**

$$v(t) = \frac{d\varphi(t)}{dt} \qquad i(t) = \frac{dq(t)}{dt}$$

$$q(t) = \int_{-\infty}^{t} i(t)dt \qquad \varphi(t) = \int_{-\infty}^{t} v(t)dt$$

INTRODUCTION

Basic electrical elements in circuit theory



Schematic representation in network analysis and synthesis

- **Components:** Resistance, capacitor, and inductor.
- **Interest variables:** voltage *v*(*t*), current *i*(*t*), el magnetic flux φ(*t*), and electrical charge *q*(*t*).
- **Relationships:**

$$v(t) = \frac{d\varphi(t)}{dt} \qquad i(t) = \frac{dq(t)}{dt}$$

$$q(t) = \int_{-\infty}^{t} i(t)dt \qquad \varphi(t) = \int_{-\infty}^{t} v(t)dt$$

- Analysis of constitutive equations and aspects of **linearity and time-variance.**

RESISTANCE

v-i characteristic: From linear to nonlinear concepts

RESISTANCE

v-i characteristic: From linear to nonlinear concepts

- **Most familiar circuit element** that one encounters in physics or in an elementary electrical engineering.

RESISTANCE

v-i characteristic: From linear to nonlinear concepts

- **Most familiar circuit element** that one encounters in physics or in an elementary electrical engineering.
  - Linear version: A two-terminal resistor satisfies Ohm's law.

RESISTANCE

v-i characteristic: From linear to nonlinear concepts

- **Most familiar circuit element** that one encounters in physics or in an elementary electrical engineering.

  - Linear version: A two-terminal resistor satisfies Ohm's law.

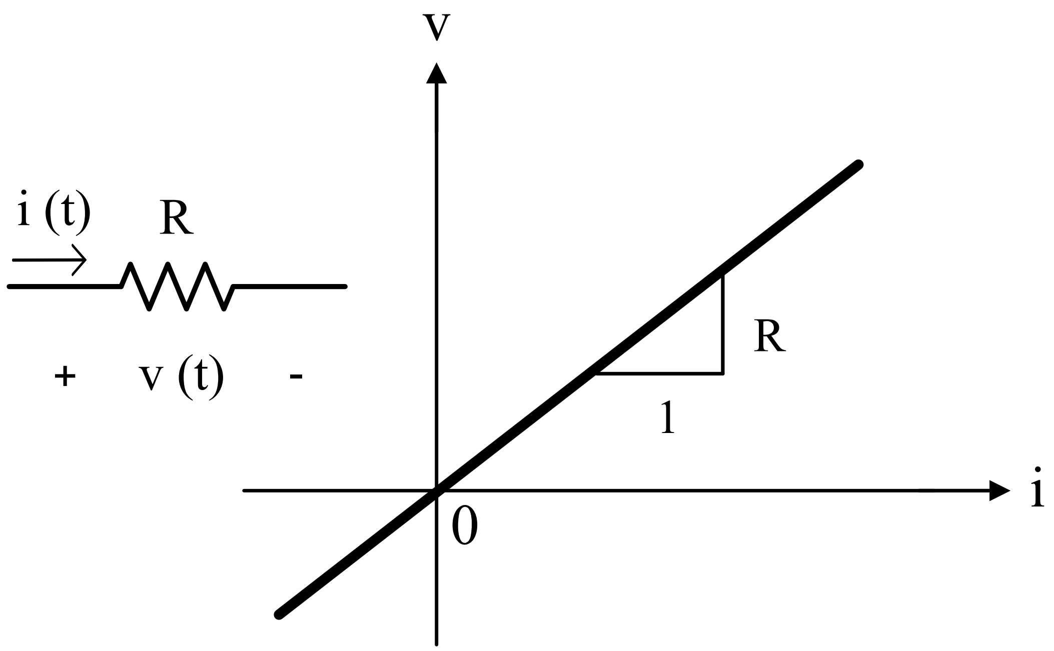
$v(t) = Ri(t)$ or $i(t) = Gv(t)$

RESISTANCE

v-i characteristic: From linear to nonlinear concepts

- **Most familiar circuit element** that one encounters in physics or in an elementary electrical engineering.
  - Linear version: A two-terminal resistor satisfies Ohm's law.

$v(t) = Ri(t)$ or $i(t) = Gv(t)$



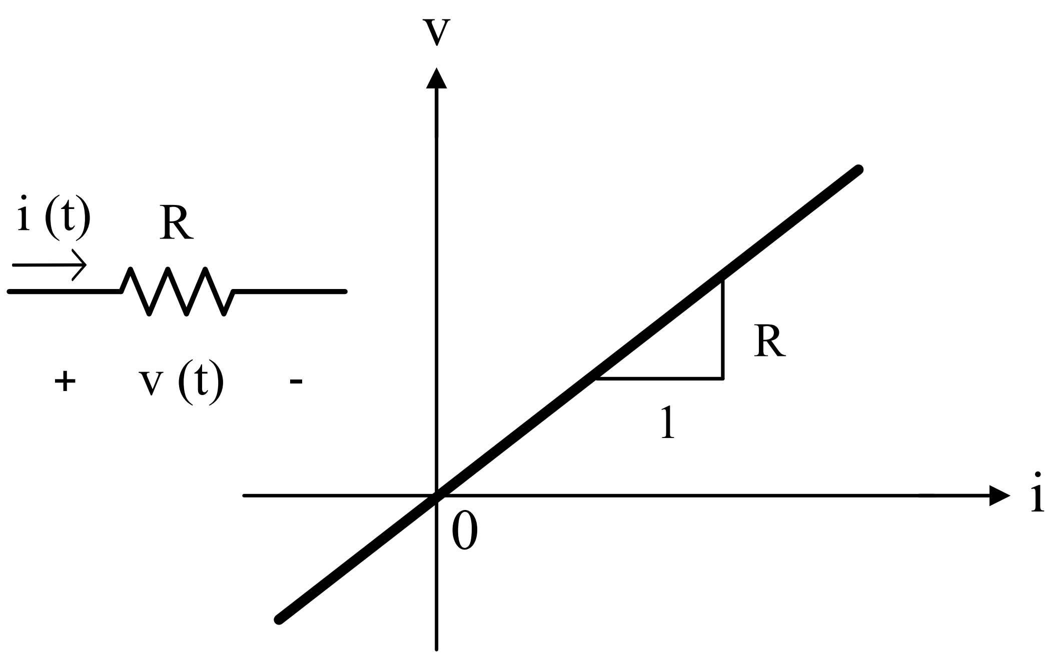
RESISTANCE

v-i characteristic: From linear to nonlinear concepts

- **Most familiar circuit element** that one encounters in physics or in an elementary electrical engineering.

  - Linear version: A two-terminal resistor satisfies Ohm's law.

$$v(t) = Ri(t) \qquad \text{or} \qquad i(t) = Gv(t)$$



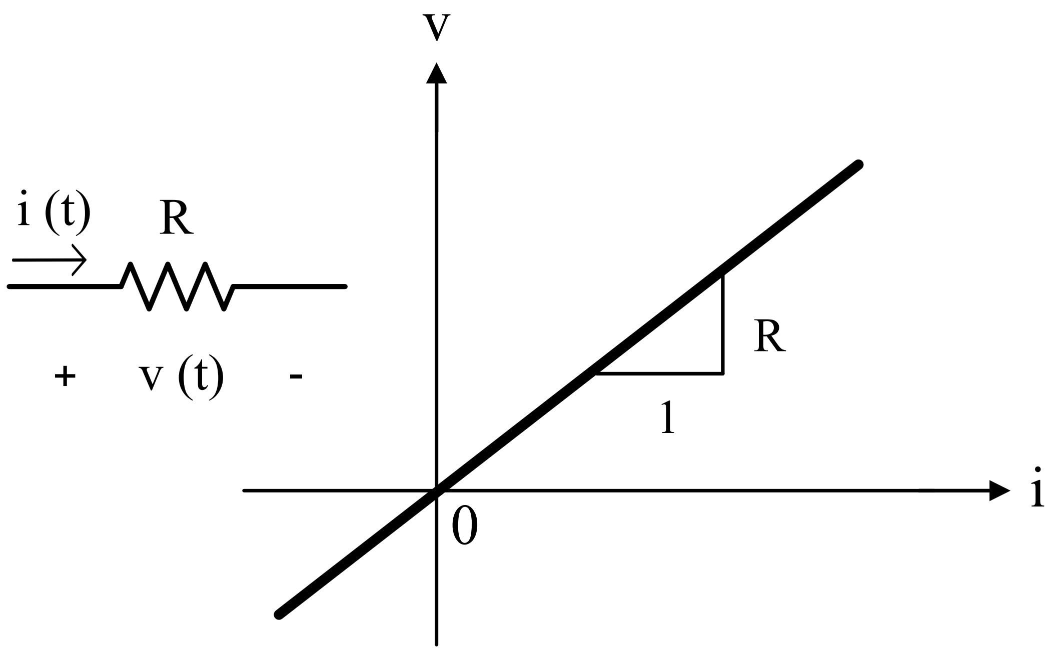
  - Nonlinear and time-variant case:

RESISTANCE

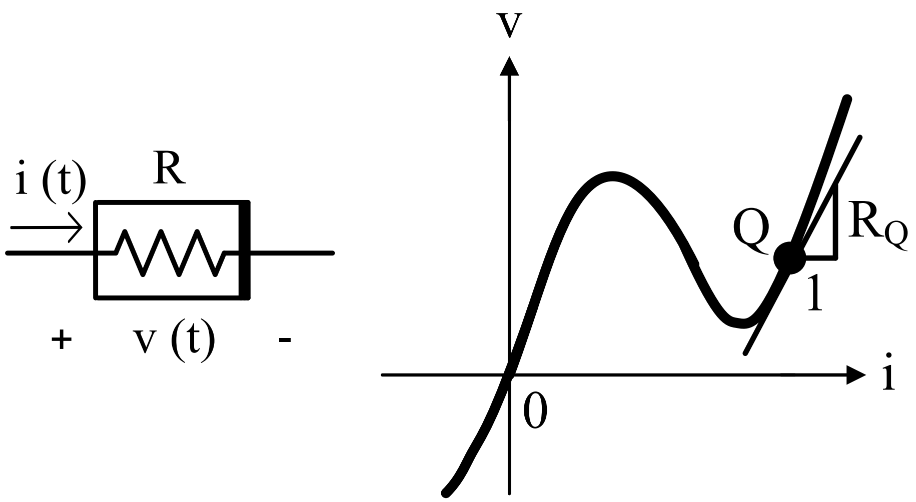
v-i characteristic: From linear to nonlinear concepts

- **Most familiar circuit element** that one encounters in physics or in an elementary electrical engineering.
  - Linear version: A two-terminal resistor satisfies Ohm's law.

$$v(t) = Ri(t) \qquad \text{or} \qquad i(t) = Gv(t)$$



  - Nonlinear and time-variant case:



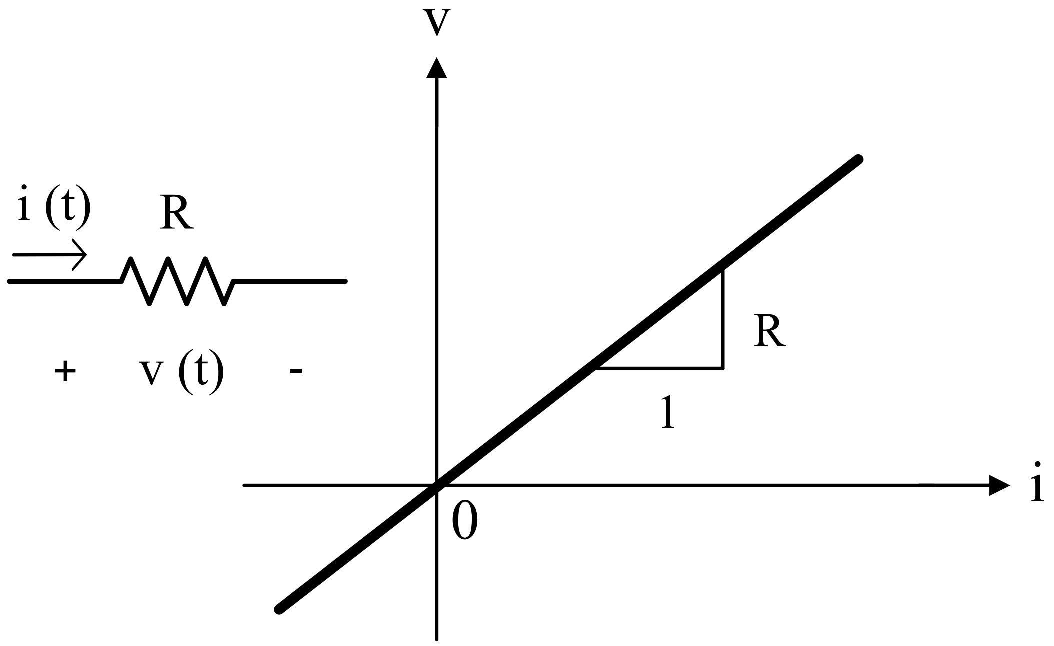
RESISTANCE

v-i characteristic: From linear to nonlinear concepts

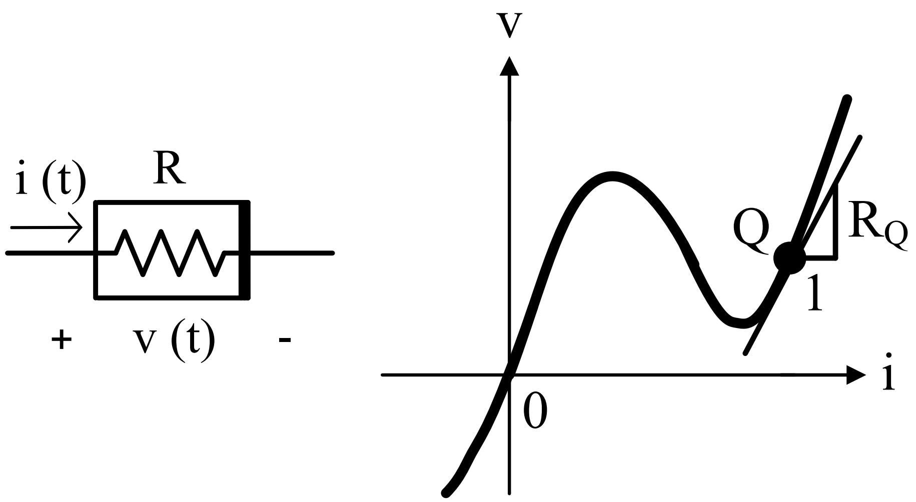
- **Most familiar circuit element** that one encounters in physics or in an elementary electrical engineering.

  - Linear version: A two-terminal resistor satisfies Ohm's law.

$v(t) = Ri(t)$ or $i(t) = Gv(t)$



  - Nonlinear and time-variant case:



➢ *Incremental or small-signal resistance*: $R_Q = \frac{dv}{di}|_Q$

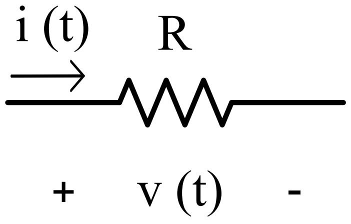
RESISTANCE

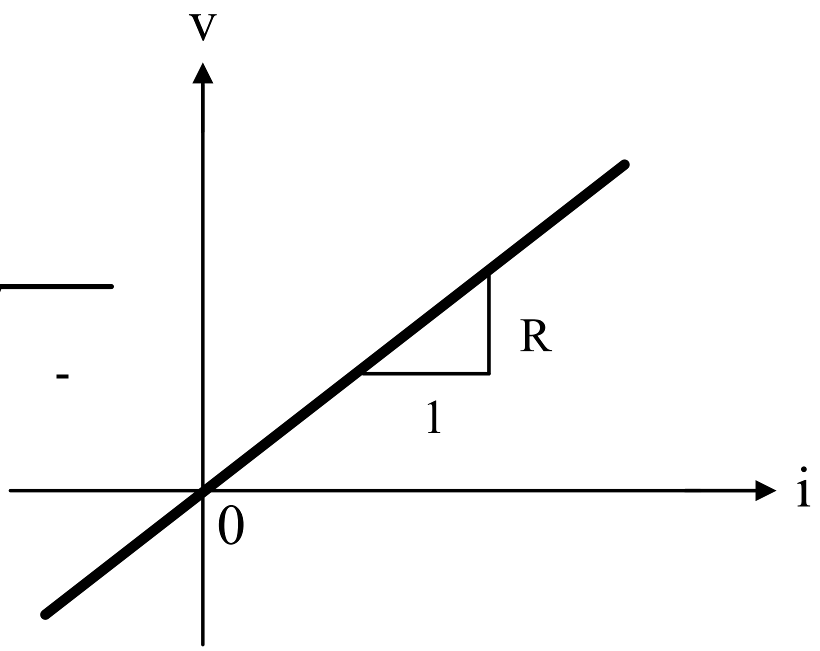
v-i characteristic: From linear to nonlinear concepts

- **Most familiar circuit element** that one encounters in physics or in an elementary electrical engineering.

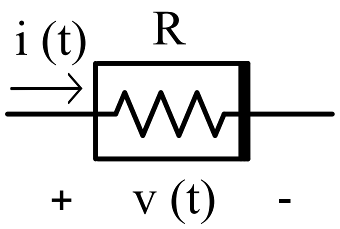
  - Linear version: A two-terminal resistor satisfies Ohm's law.

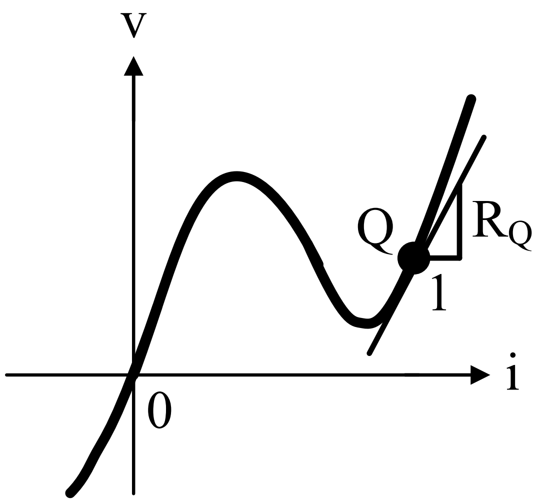
$v(t) = Ri(t)$ or $i(t) = Gv(t)$





  - Nonlinear and time-variant case:





- *Incremental or small-signal resistance*: $R_Q = \frac{dv}{di}|_Q$
- Example: *negative resistance?, control in current but not in voltage.*

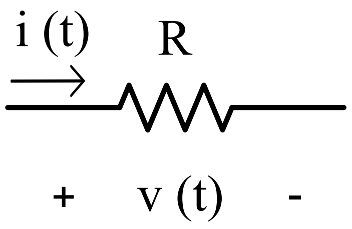
RESISTANCE

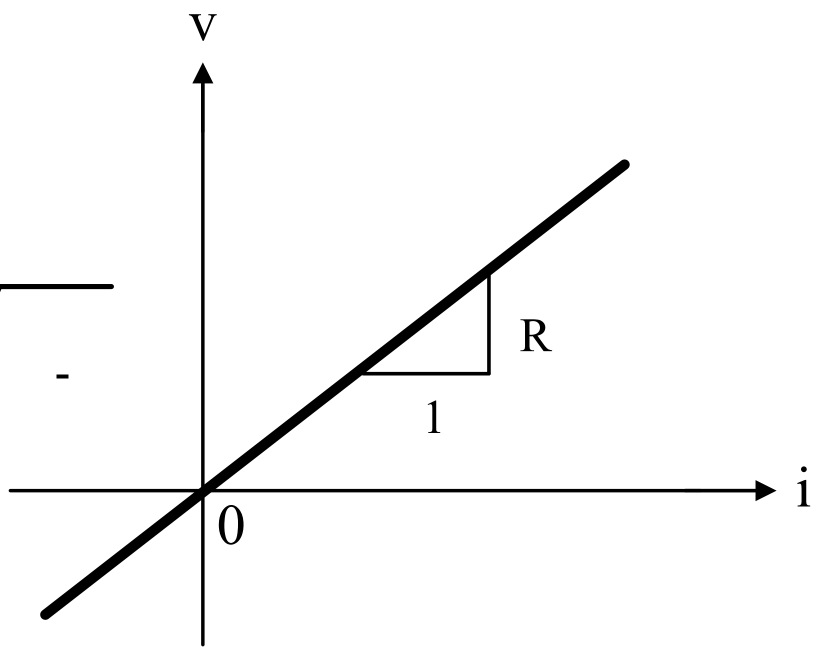
v-i characteristic: From linear to nonlinear concepts

- **Most familiar circuit element** that one encounters in physics or in an elementary electrical engineering.

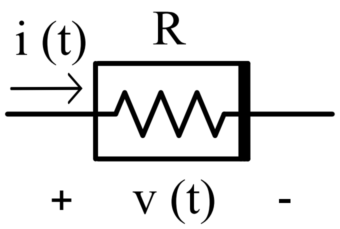
  - Linear version: A two-terminal resistor satisfies Ohm's law.

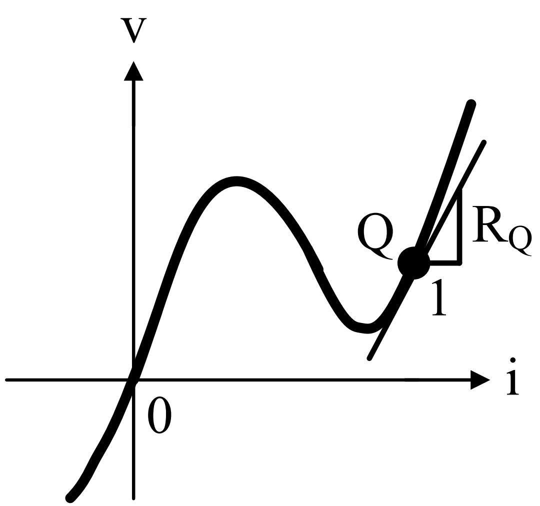
$v(t) = Ri(t)$ or $i(t) = Gv(t)$





  - Nonlinear and time-variant case:





➢ *Incremental or small-signal resistance:* $R_Q = \frac{dv}{di}|_Q$

➢ Example: *negative resistance?, control in current but not in voltage.*

➢ *Feature of time-variance:* $f_R(v, i, t) = 0$

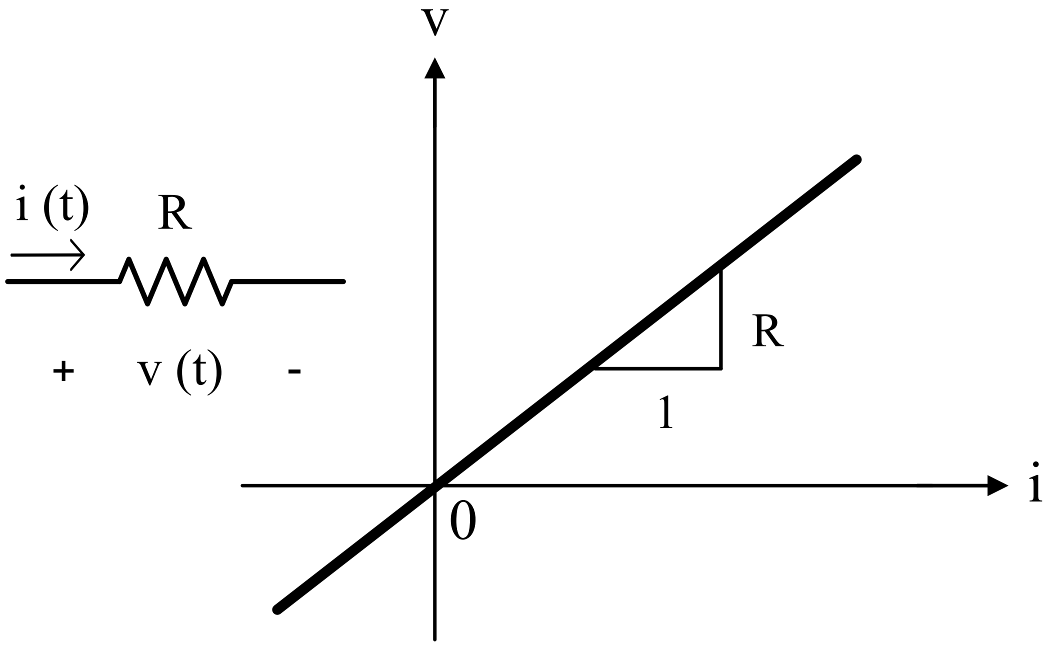
RESISTANCE

v-i characteristic: From linear to nonlinear concepts

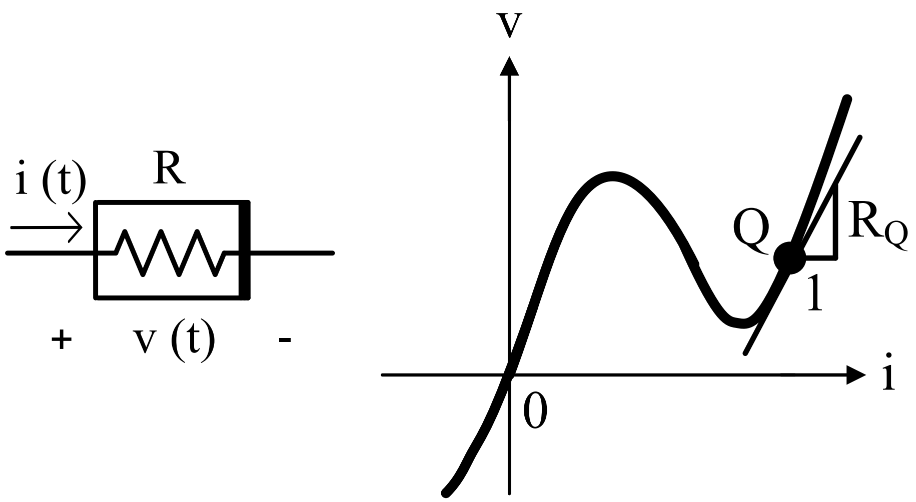
- **Most familiar circuit element** that one encounters in physics or in an elementary electrical engineering.

  - Linear version: A two-terminal resistor satisfies Ohm's law.

$v(t) = Ri(t)$ or $i(t) = Gv(t)$



  - Nonlinear and time-variant case:



➢ *Incremental or small-signal resistance:* $R_Q = \frac{dv}{di}|_Q$

➢ Example: *negative resistance?, control in current but not in voltage.*

➢ *Feature of time-variance:* $f_R(v, i, t) = 0$

Linear and time-invariant v-i curve $f_R(v, i) = v - Ri$

CAPACITOR

q-v characteristic

CAPACITOR

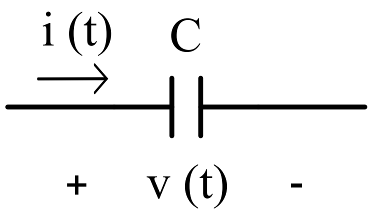
q-v characteristic

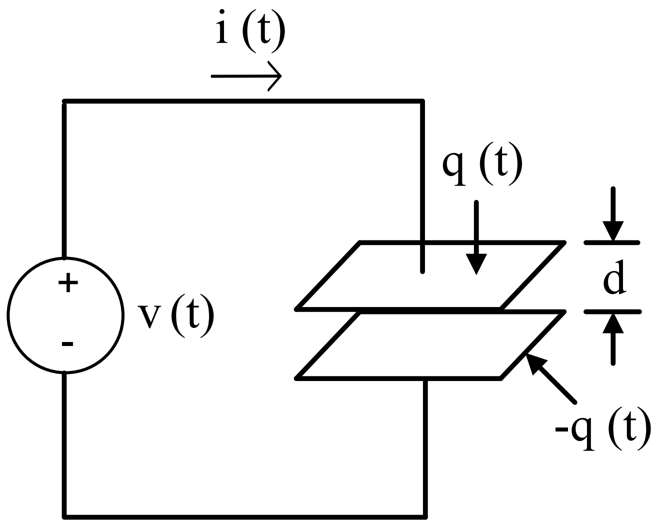
- A two-terminal element whose charge $q(t)$ and voltage $v(t)$ fall on some fixed curve in the q-v plane at any time t is called **a time-invariant capacitor.**

CAPACITOR

q-v characteristic

- A two-terminal element whose charge $q(t)$ and voltage $v(t)$ fall on some fixed curve in the q-v plane at any time t is called **a time-invariant capacitor.**



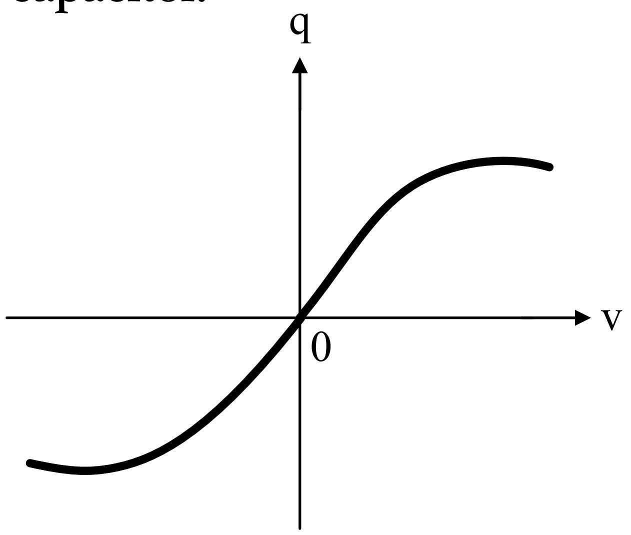


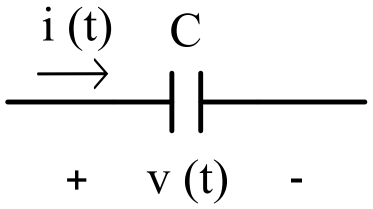
CAPACITOR

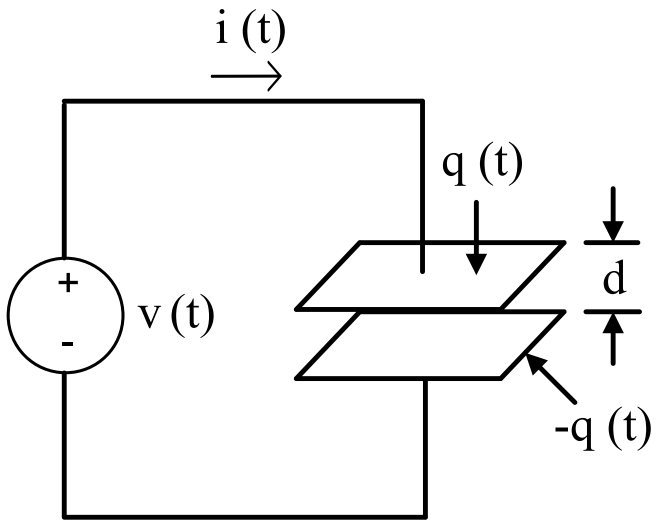
q-v characteristic

- A two-terminal element whose charge *q*(*t*) and voltage *v*(*t*) fall on some fixed curve in the *q*-*v* plane at any time *t* is called **a time-invariant capacitor.**

- **Non-linear time-invariant capacitor**



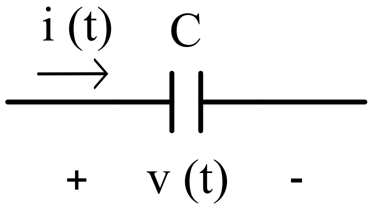


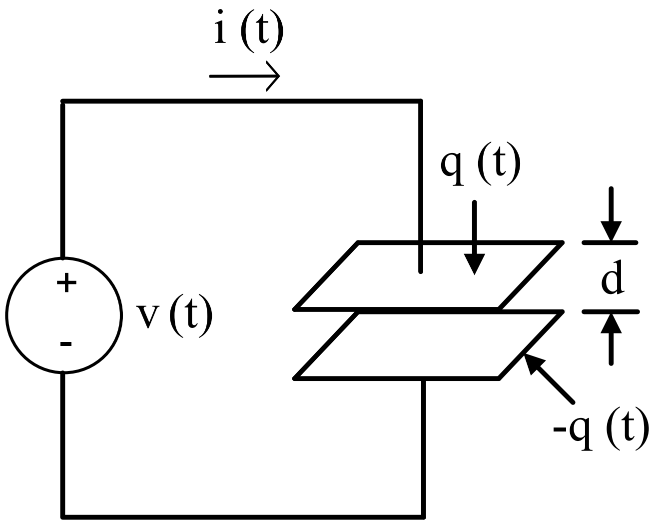


CAPACITOR

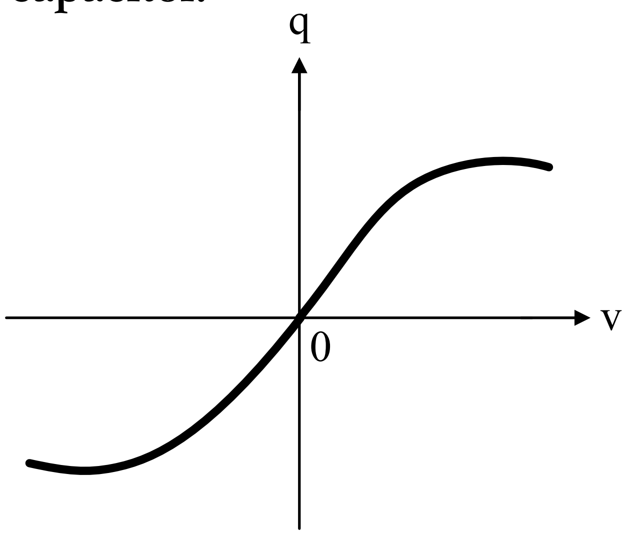
q-v characteristic

- A two-terminal element whose charge $q(t)$ and voltage $v(t)$ fall on some fixed curve in the q-v plane at any time t is called **a time-invariant capacitor.**





- **Non-linear time-invariant capacitor**



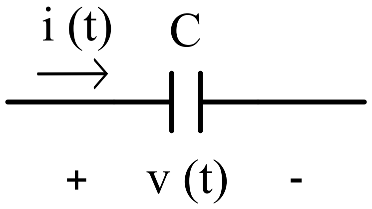
➢ *Concept of incremental or small-signal capacitor*:

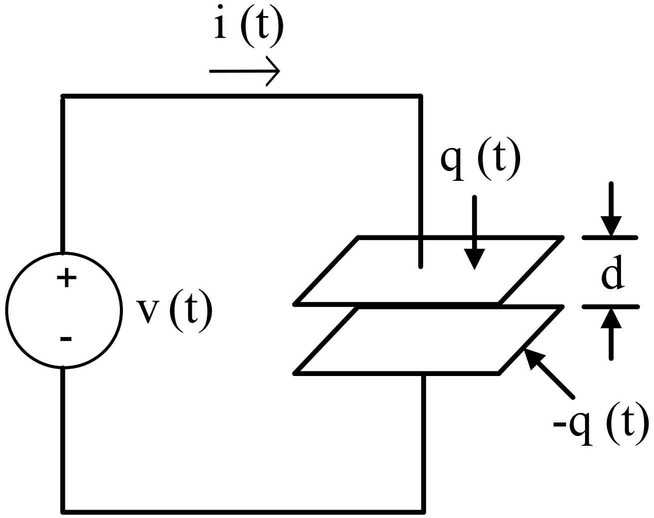
$$C_Q = \frac{dq}{dv}\Big|_Q$$

CAPACITOR

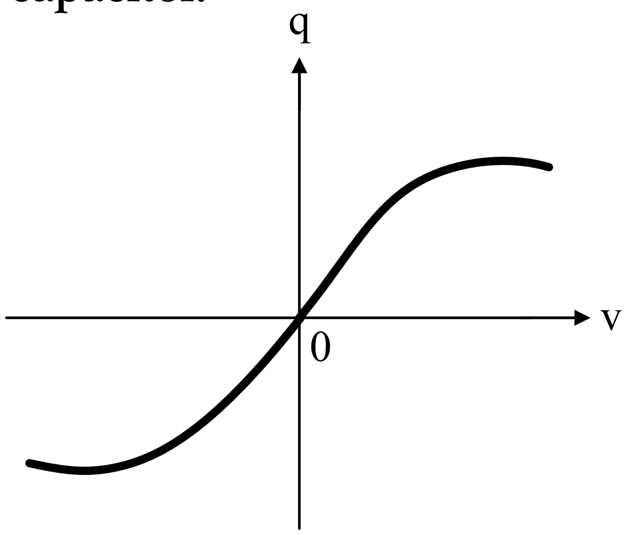
q-v characteristic

- A two-terminal element whose charge $q(t)$ and voltage $v(t)$ fall on some fixed curve in the q-v plane at any time t is called **a time-invariant capacitor.**





- **Non-linear time-invariant capacitor**



➢ *Concept of incremental or small-signal capacitor*:

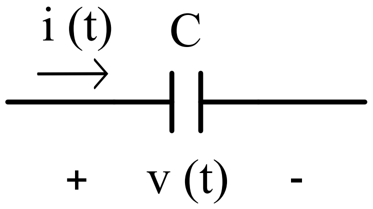
$$C_Q = \frac{dq}{dv}\Big|_Q$$

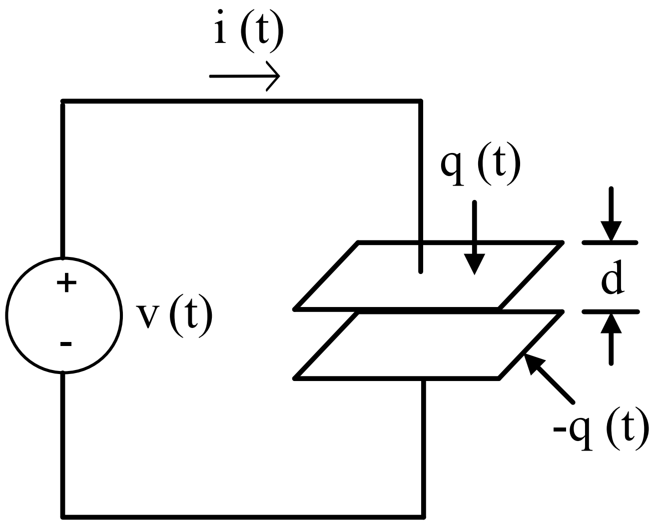
- **Time-varying capacitors**

CAPACITOR

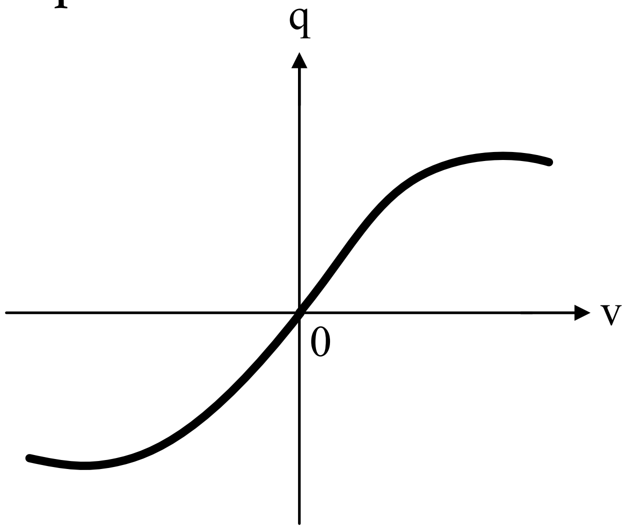
q-v characteristic

- A two-terminal element whose charge $q(t)$ and voltage $v(t)$ fall on some fixed curve in the q-v plane at any time t is called **a time-invariant capacitor.**





- **Non-linear time-invariant capacitor**

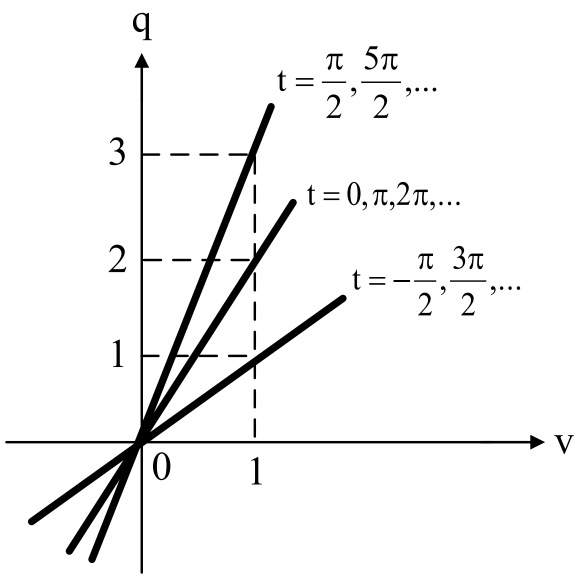


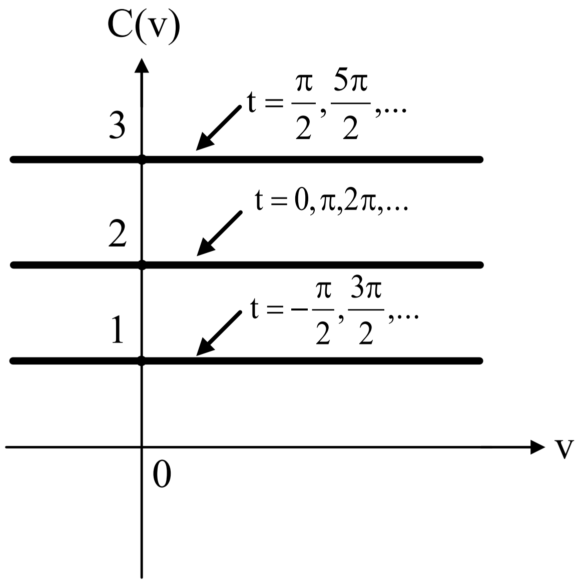
➢ *Concept of incremental or small-signal capacitor*:

$$C_Q = \frac{dq}{dv}\Big|_Q$$

- **Time-varying capacitors**

➢ Examples:

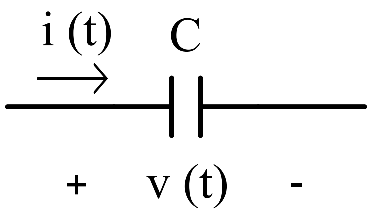


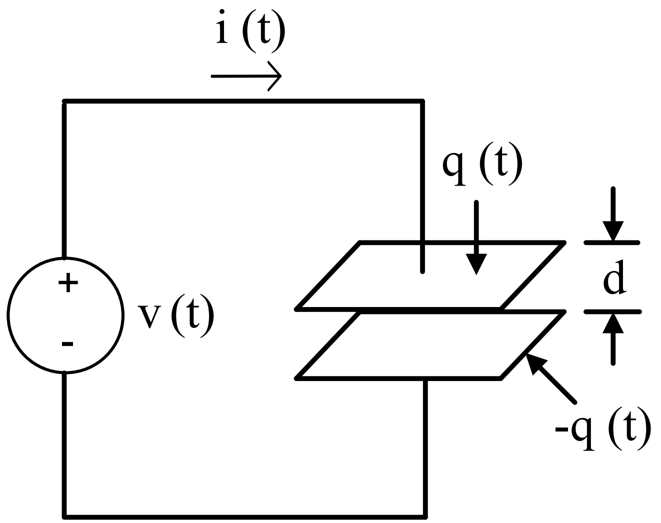


CAPACITOR

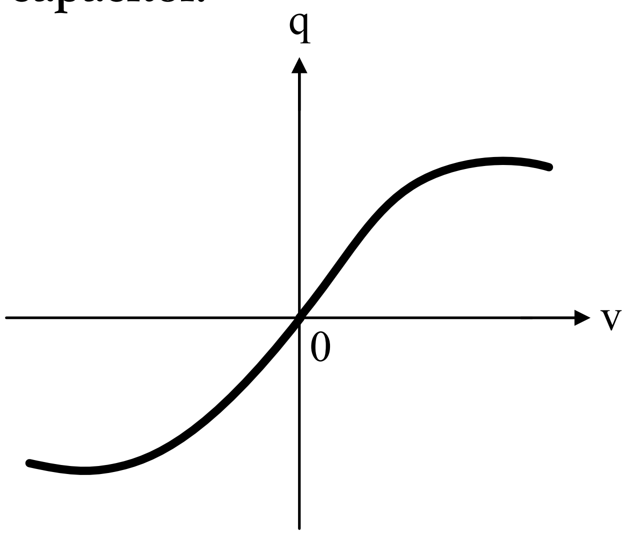
q-v characteristic

- A two-terminal element whose charge $q(t)$ and voltage $v(t)$ fall on some fixed curve in the q-v plane at any time t is called **a time-invariant capacitor.**





- **Non-linear time-invariant capacitor**



➢ *Concept of incremental or small-signal capacitor*:

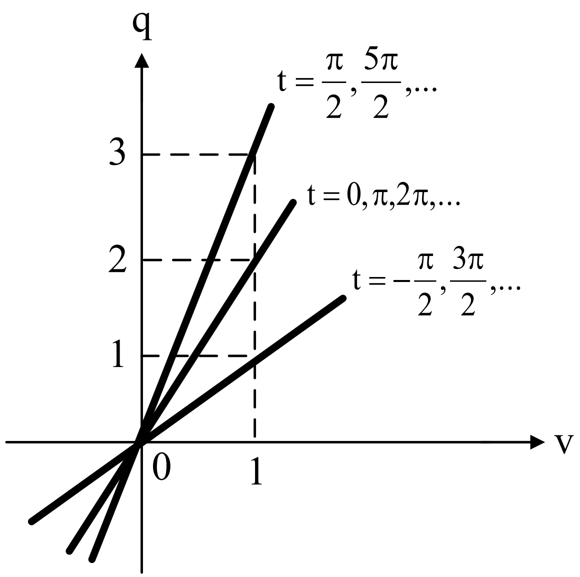
$$C_Q = \frac{dq}{dv}\Big|_Q$$

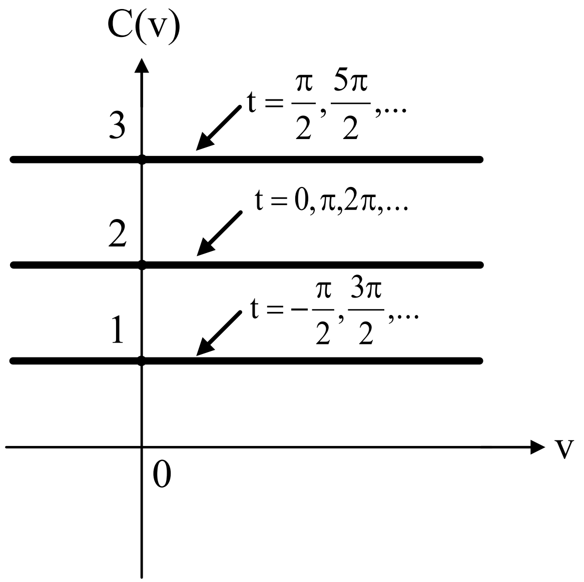
$$f_C(q, v, t) = q - Cv$$

- **Time-varying capacitors**

$$i(t) = C(t)\frac{dv(t)}{dt} + v(t)\frac{dC(t)}{dt}$$

➢ Examples:

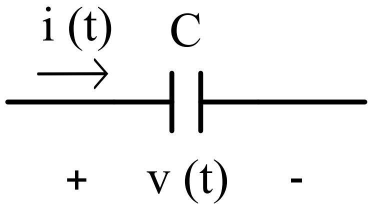


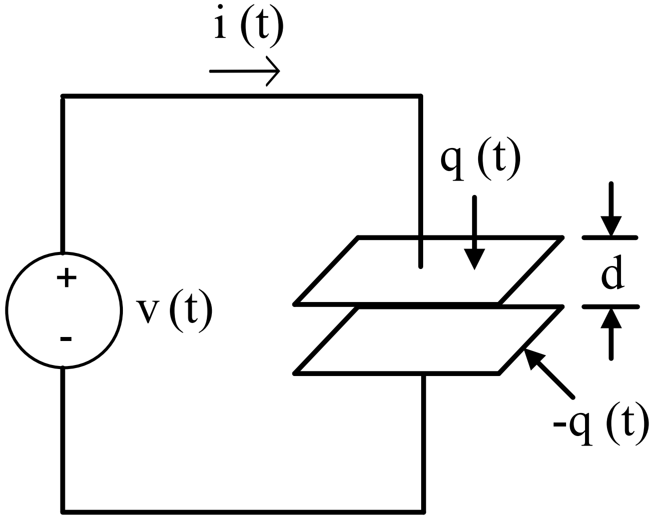


CAPACITOR

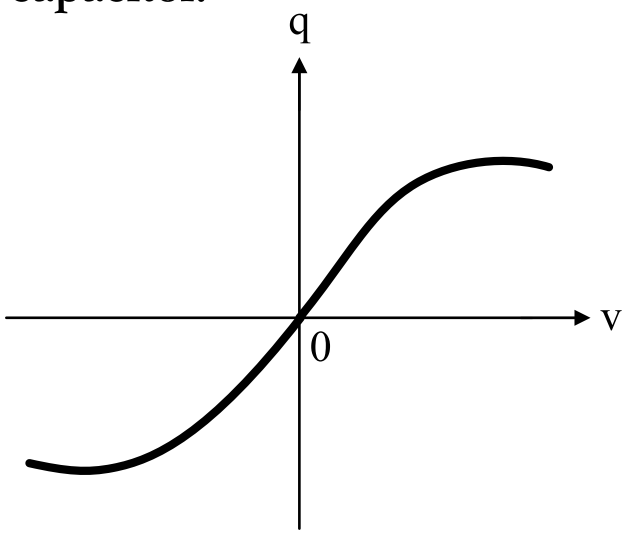
q-v characteristic

- A two-terminal element whose charge $q(t)$ and voltage $v(t)$ fall on some fixed curve in the q-v plane at any time t is called **a time-invariant capacitor.**





- **Non-linear time-invariant capacitor**



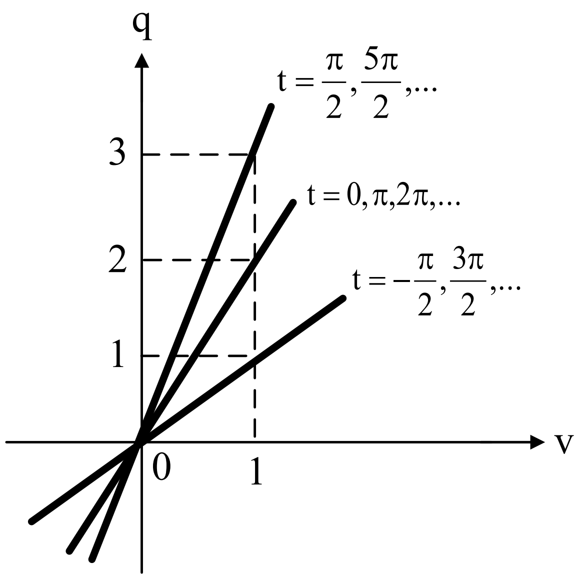
➢ *Concept of incremental or small-signal capacitor*:

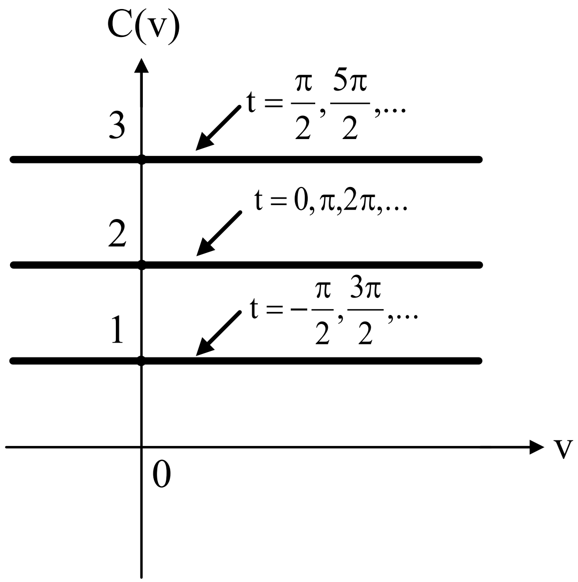
$$C_Q = \frac{dq}{dv}\Big|_Q$$

$$f_C(q, v, t) = q - Cv$$

- **Time-varying capacitors**

$$i(t) = C(t)\frac{dv(t)}{dt} + v(t)\frac{dC(t)}{dt}$$





➢ Examples:

Linear and time-invariant capacitor

$$i(t) = C\frac{dv(t)}{dt}$$

INDUCTOR

φ-i characteristic

INDUCTOR

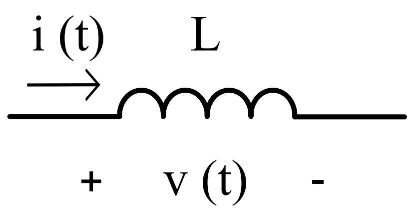
φ-i characteristic

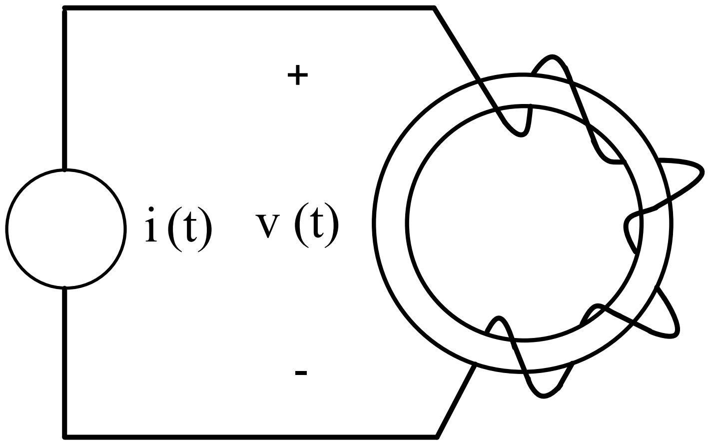
- A two-terminal element whose flux $\varphi(t)$ and current $i(t)$ fall on some fixed curve in the φ-i plane at any time t is called **a time-invariant inductor.**

INDUCTOR

φ-i characteristic

- A two-terminal element whose flux $\varphi(t)$ and current $i(t)$ fall on some fixed curve in the φ-i plane at any time t is called **a time-invariant inductor.**



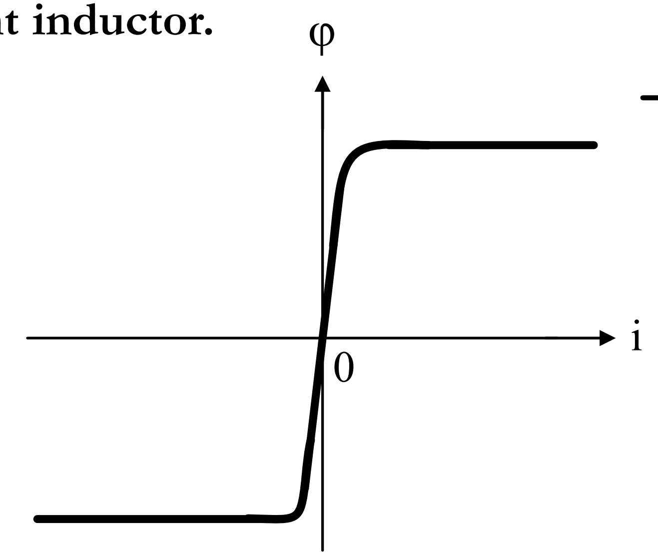


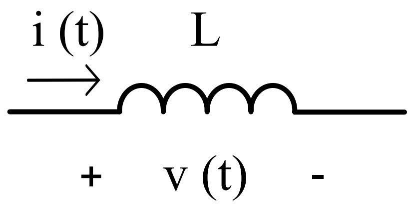
INDUCTOR

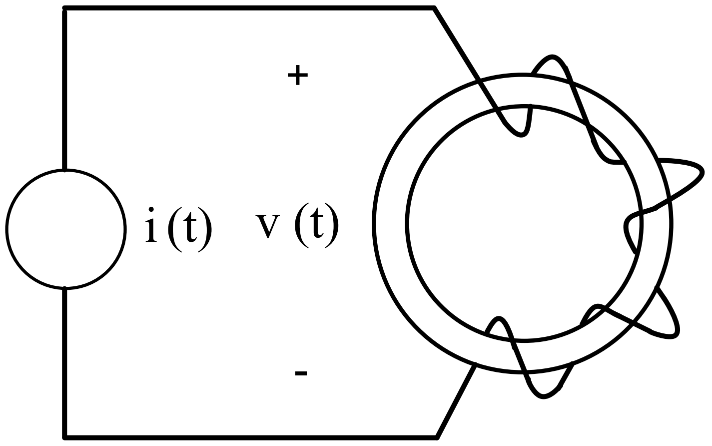
φ-i characteristic

- A two-terminal element whose flux φ(*t*) and current *i*(*t*) fall on some fixed curve in the φ-i plane at any time *t* is called **a time-invariant inductor.**

- **Non-linear time-invariant inductor**



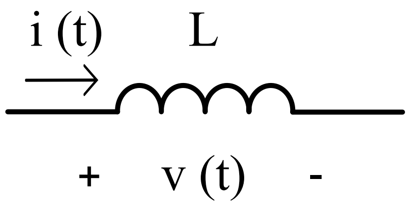


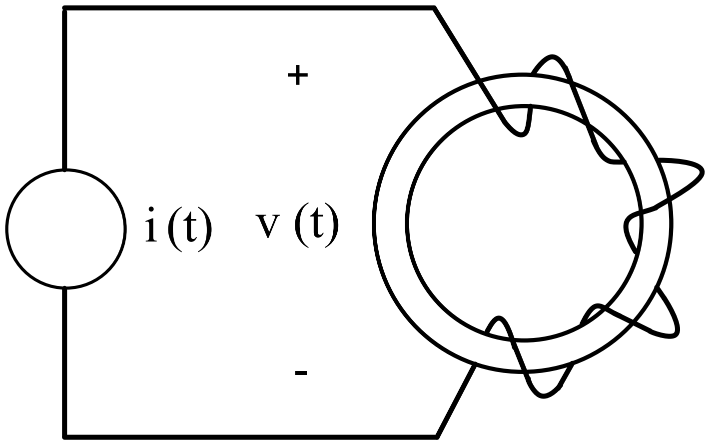


INDUCTOR

φ-i characteristic

- A two-terminal element whose flux $\varphi(t)$ and current $i(t)$ fall on some fixed curve in the φ-i plane at any time t is called **a time-invariant inductor.**

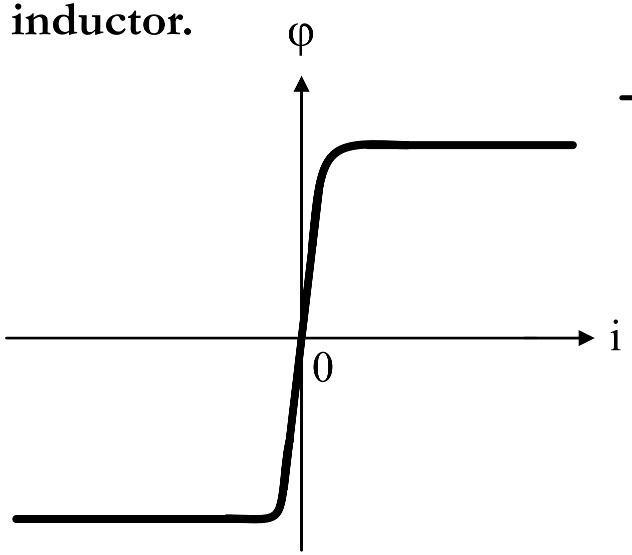




- **Non-linear time-invariant inductor**

➢ *Concept of incremental or small-signal inductor:*

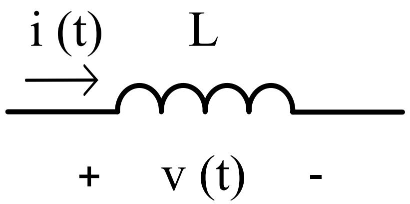
$$L_Q = \frac{d\varphi}{di}\Big|_Q$$

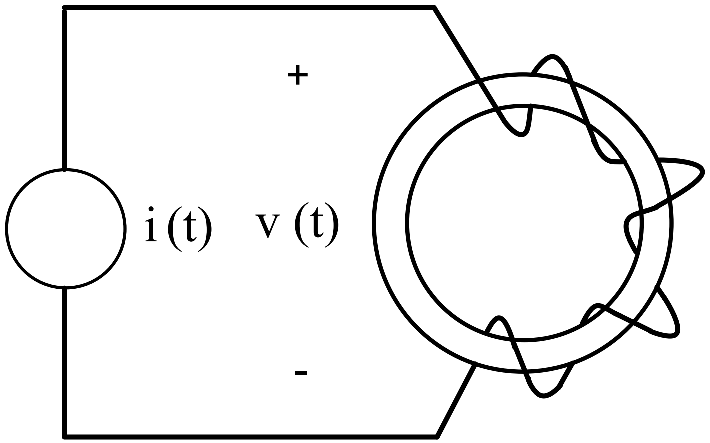


INDUCTOR

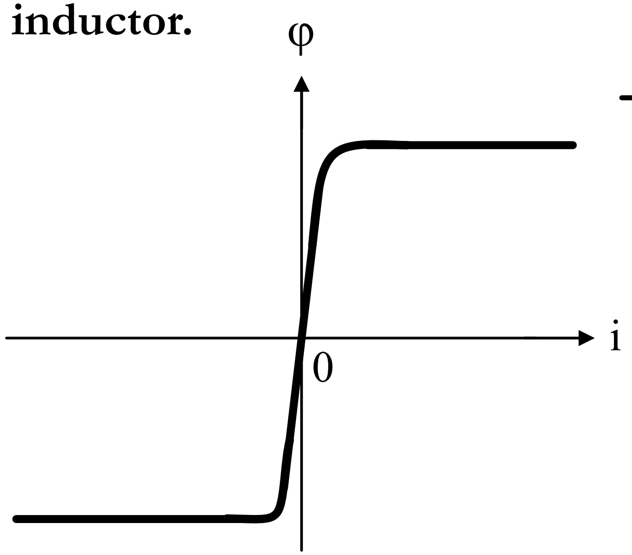
φ-i characteristic

- A two-terminal element whose flux $\varphi(t)$ and current $i(t)$ fall on some fixed curve in the φ-i plane at any time t is called **a time-invariant inductor.**





- **Non-linear time-invariant inductor**



➢ *Concept of incremental or small-signal inductor*:

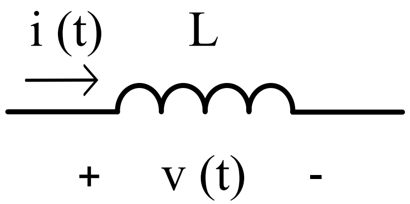
$$L_Q = \frac{d\varphi}{di}\Big|_Q$$

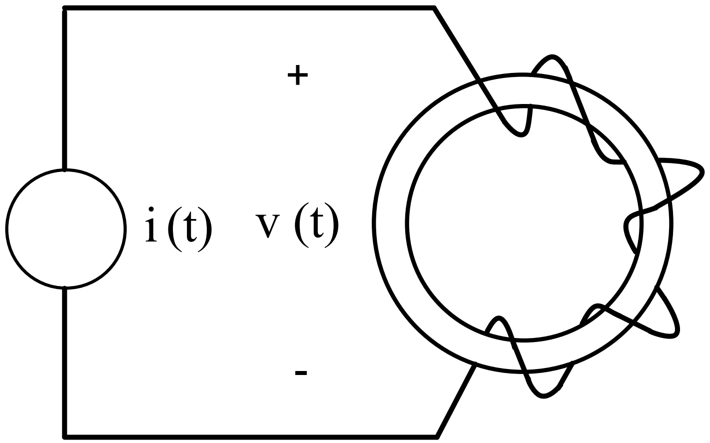
- **Time-varying inductors**

INDUCTOR

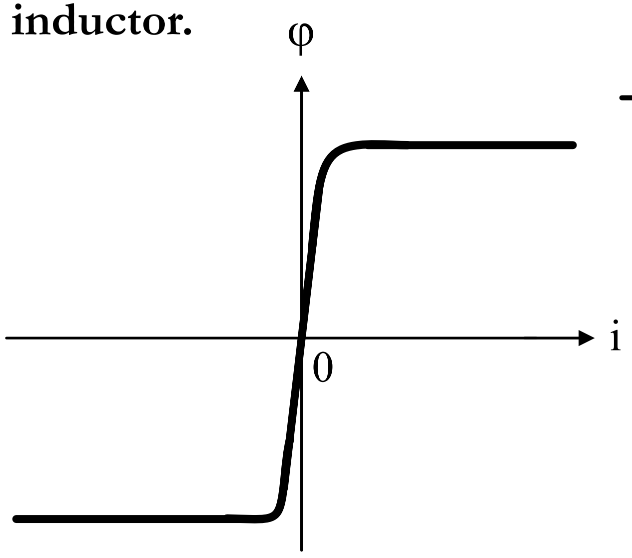
φ-i characteristic

- A two-terminal element whose flux $\varphi(t)$ and current $i(t)$ fall on some fixed curve in the φ-i plane at any time t is called **a time-invariant inductor.**





- **Non-linear time-invariant inductor**

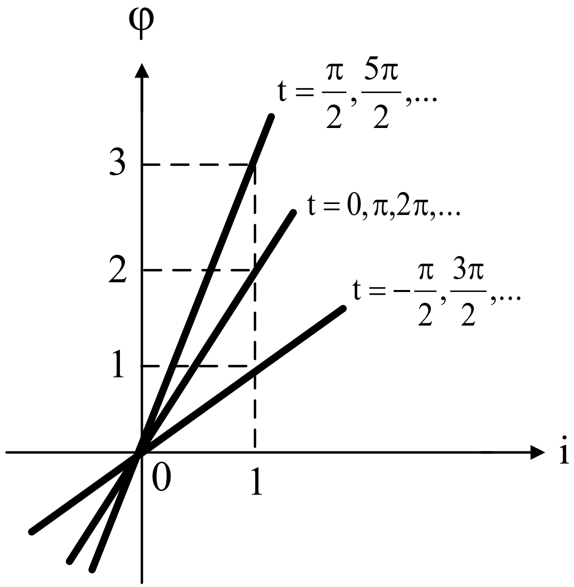


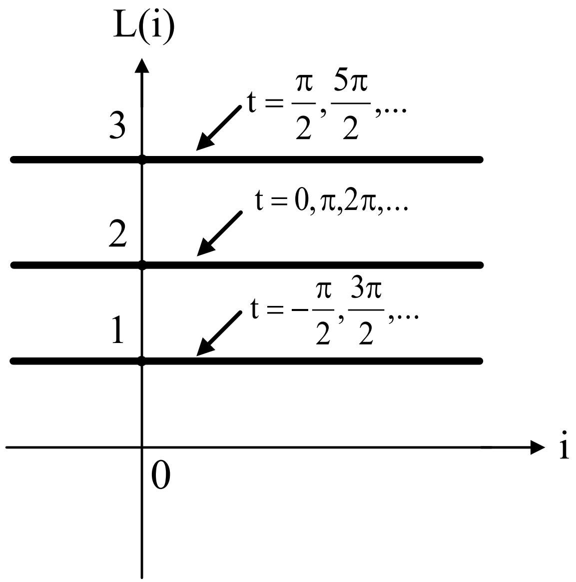
➢ *Concept of incremental or small-signal inductor*:

$$L_Q = \frac{d\varphi}{di}|_Q$$

- **Time-varying inductors**

➢ Examples:

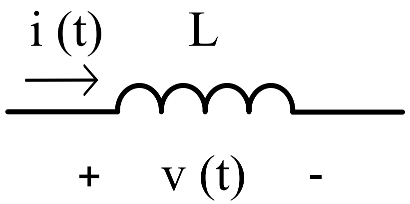


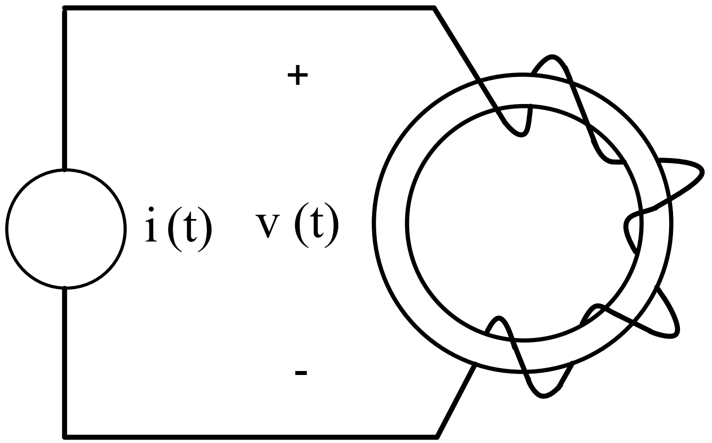


INDUCTOR

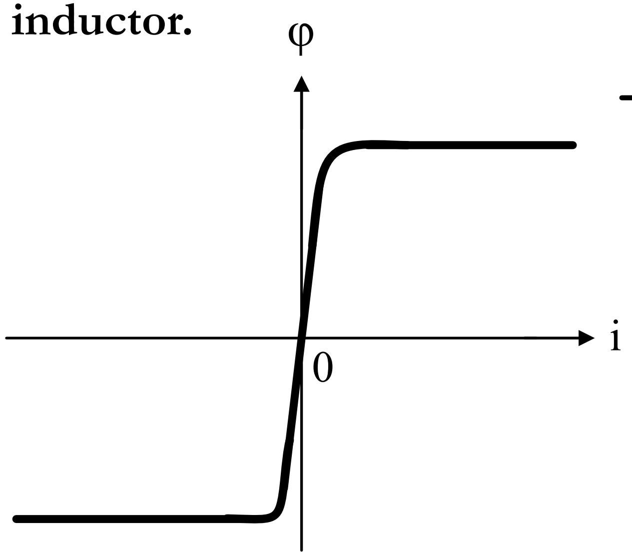
φ-i characteristic

- A two-terminal element whose flux $\varphi(t)$ and current $i(t)$ fall on some fixed curve in the φ-i plane at any time t is called **a time-invariant inductor.**





- **Non-linear time-invariant inductor**



➢ *Concept of incremental or small-signal inductor*:

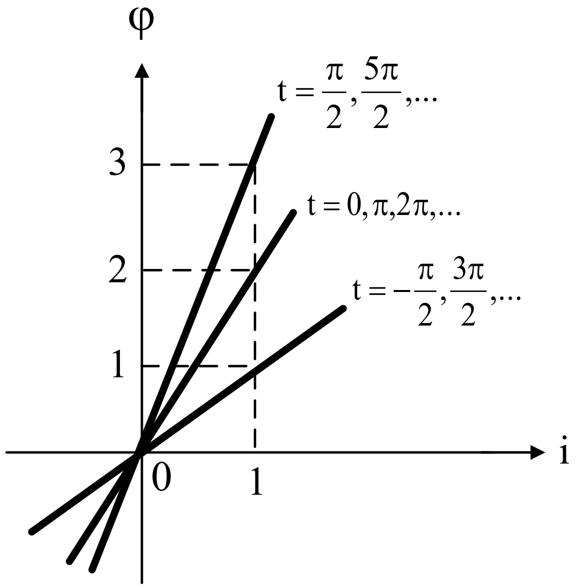
$$L_Q = \frac{d\varphi}{di}\Big|_Q$$

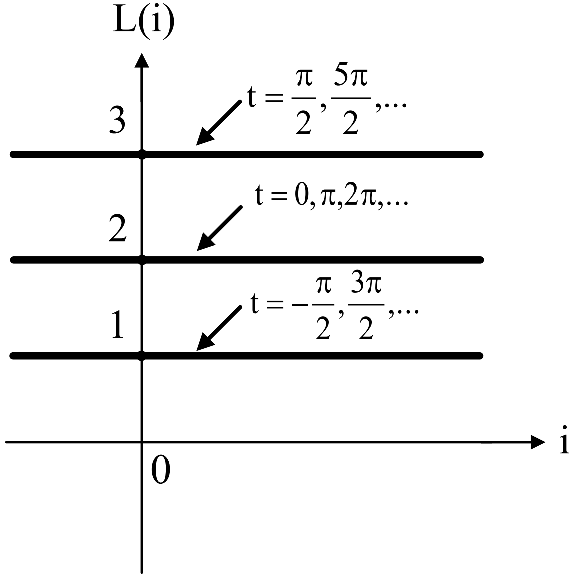
$$f_L(\varphi, i, t) = \varphi - Li$$

- **Time-varying inductors**

$$v(t) = L(t)\frac{di(t)}{dt} + i(t)\frac{dL(t)}{dt}$$

➢ Examples:

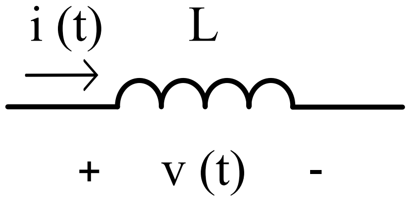


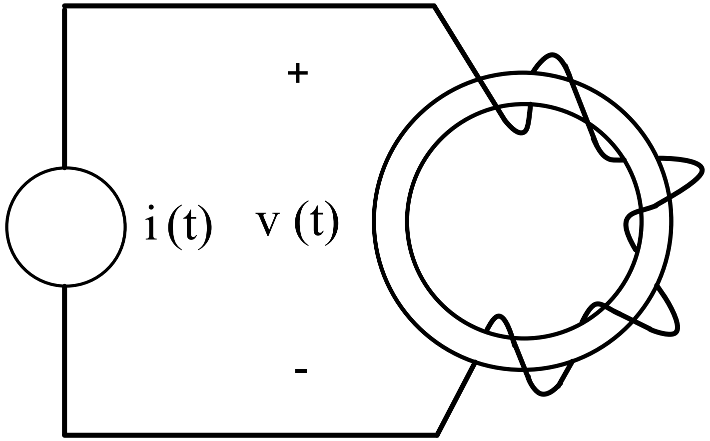


INDUCTOR

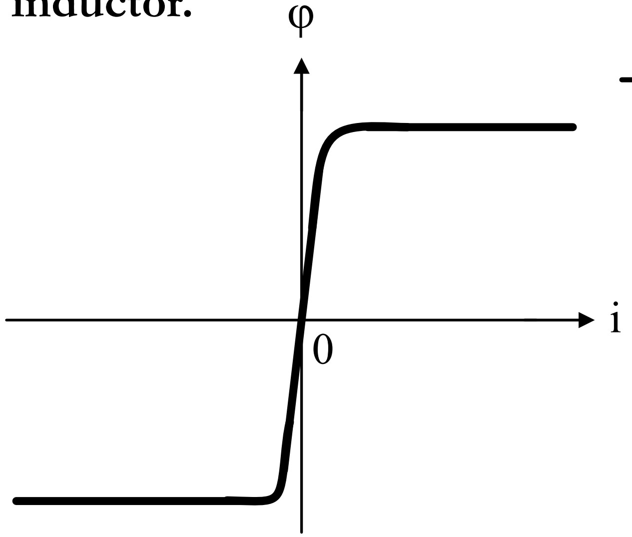
φ-i characteristic

- A two-terminal element whose flux $\varphi(t)$ and current $i(t)$ fall on some fixed curve in the φ-i plane at any time t is called **a time-invariant inductor.**





- **Non-linear time-invariant inductor**



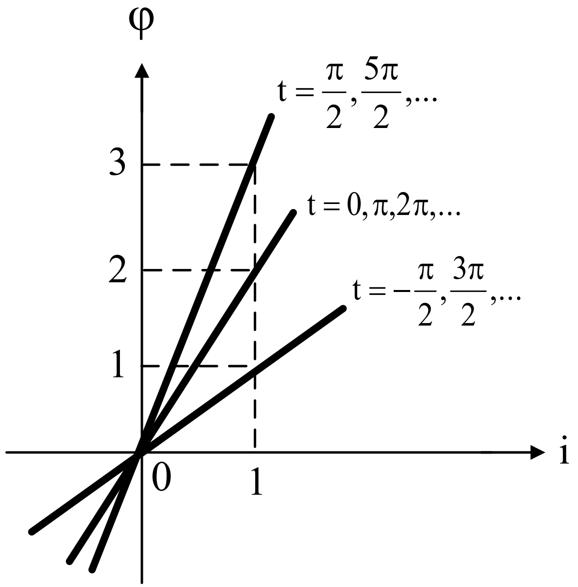
➢ *Concept of incremental or small-signal inductor*:

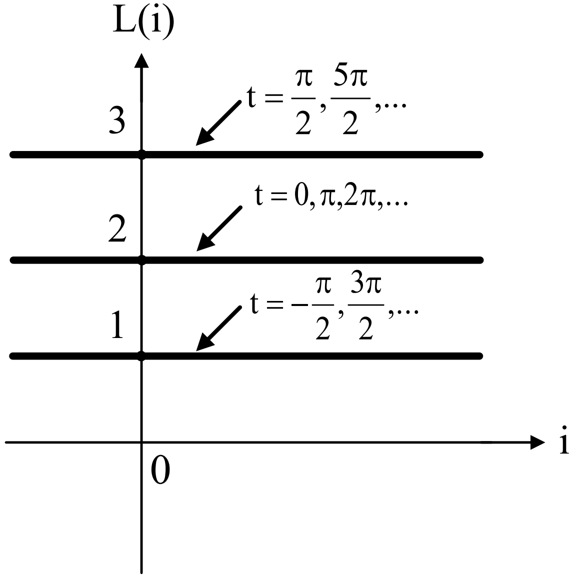
$$L_Q = \frac{d\varphi}{di}\Big|_Q$$

$$f_L(\varphi, i, t) = \varphi - Li$$

- **Time-varying inductors**

$$v(t) = L(t)\frac{di(t)}{dt} + i(t)\frac{dL(t)}{dt}$$





➢ Examples:

Linear and time-invariant inductor

$$v(t) = L\frac{di(t)}{dt}$$

COMPLEX IMPEDANCES

Laplace transform techniques in electrical circuits

COMPLEX IMPEDANCES

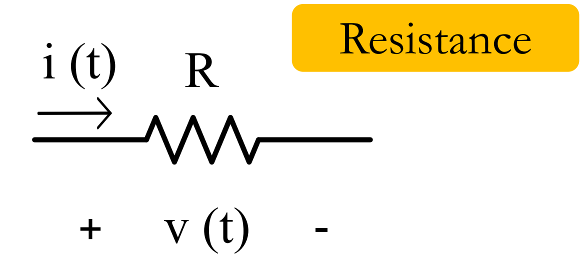
Laplace transform techniques in electrical circuits

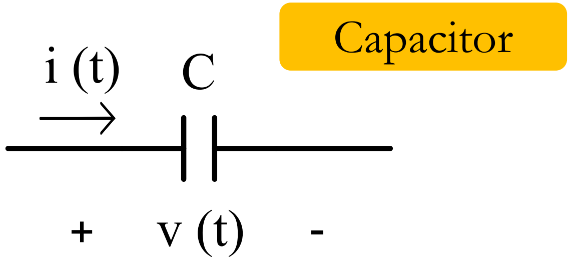
- Assumption of **LINEAR AND TIME-INVARIANT (LTI) electrical circuits**.

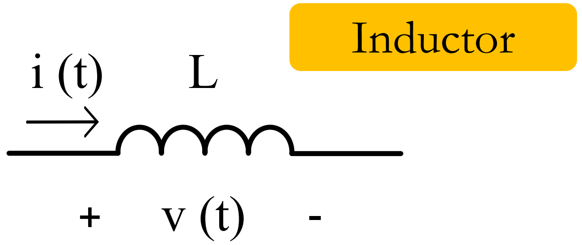
COMPLEX IMPEDANCES

Laplace transform techniques in electrical circuits

- Assumption of **LINEAR AND TIME-INVARIANT (LTI) electrical circuits**.







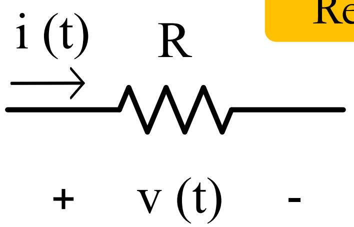
COMPLEX IMPEDANCES

Laplace transform techniques in electrical circuits

- Assumption of **LINEAR AND TIME-INVARIANT (LTI) electrical circuits**.

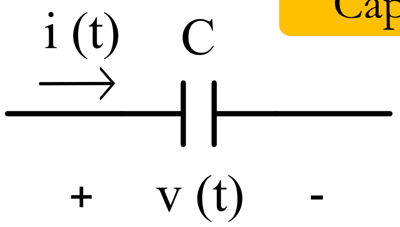
Time-domain

Resistance



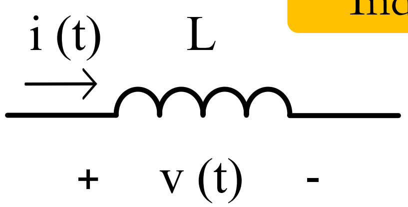
$$v(t) = Ri(t)$$

Capacitor



$$i(t) = C\frac{dv(t)}{dt}$$

Inductor



$$v(t) = L\frac{di(t)}{dt}$$

COMPLEX IMPEDANCES

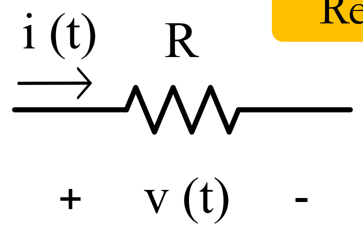
Laplace transform techniques in electrical circuits

- Assumption of **LINEAR AND TIME-INVARIANT (LTI) electrical circuits**.

Time-domain

Laplace-domain: $X(s) = \int_{-\infty}^{\infty} e^{-st} x(t) \mathrm{d}t$

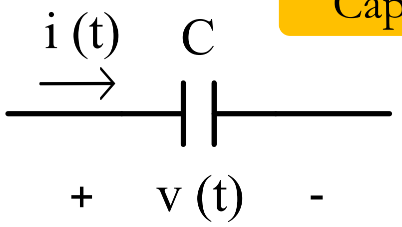
Resistance



$$v(t) = Ri(t)$$

$$Z_{\mathrm{R}}(s) = \frac{V(s)}{I(s)} = R$$

Capacitor

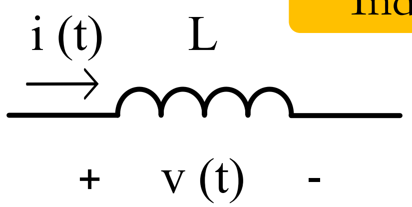


$$i(t) = C\frac{\mathrm{d}v(t)}{\mathrm{d}t}$$

$$Z_{\mathrm{C}}(s) = \frac{V(s)}{I(s)} = \frac{1}{sC}$$

$$I(s) = C[sV(s) - v(0)] \rightarrow V(s) = I(s)\frac{1}{sC} + \frac{v(0)}{s}$$

Inductor



$$v(t) = L\frac{\mathrm{d}i(t)}{\mathrm{d}t}$$

$$Z_{\mathrm{L}}(s) = \frac{V(s)}{I(s)} = sL$$

$$V(s) = L[sI(s) - i(0)] \rightarrow I(s) = \frac{V(s)}{sL} + \frac{i(0)}{s}$$

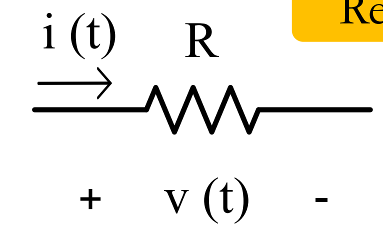
COMPLEX IMPEDANCES

Laplace transform techniques in electrical circuits

- Assumption of **LINEAR AND TIME-INVARIANT (LTI) electrical circuits**.

Time-domain *Laplace-domain*: $X(s) = \int_{-\infty}^{\infty} e^{-st}x(t)\mathrm{d}t$ *Equilant circuit with initial conditions*

Resistance

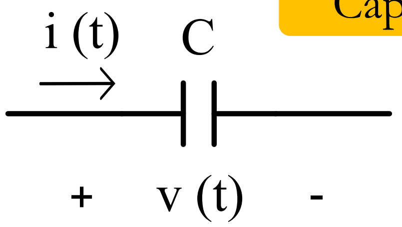


$$v(t) = Ri(t)$$

$$Z_{\mathrm{R}}(s) = \frac{V(s)}{I(s)} = R$$

−

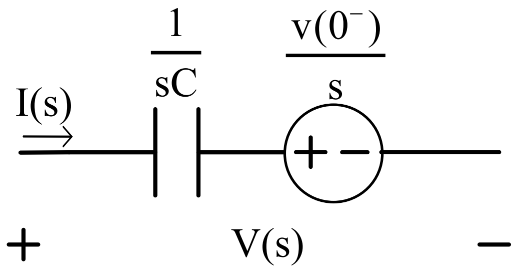
Capacitor



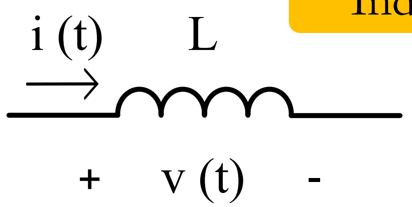
$$i(t) = C\frac{\mathrm{d}v(t)}{\mathrm{d}t}$$

$$Z_{\mathrm{C}}(s) = \frac{V(s)}{I(s)} = \frac{1}{sC}$$

$$I(s) = C[sV(s) - v(0)] \rightarrow V(s) = I(s)\frac{1}{sC} + \frac{v(0)}{s}$$



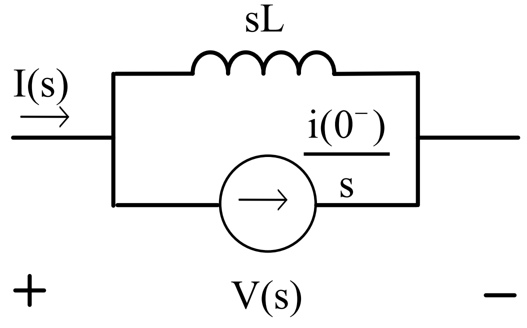
Inductor



$$v(t) = L\frac{\mathrm{d}i(t)}{\mathrm{d}t}$$

$$Z_{\mathrm{L}}(s) = \frac{V(s)}{I(s)} = sL$$

$$V(s) = L[sI(s) - i(0)] \rightarrow I(s) = \frac{V(s)}{sL} + \frac{i(0)}{s}$$

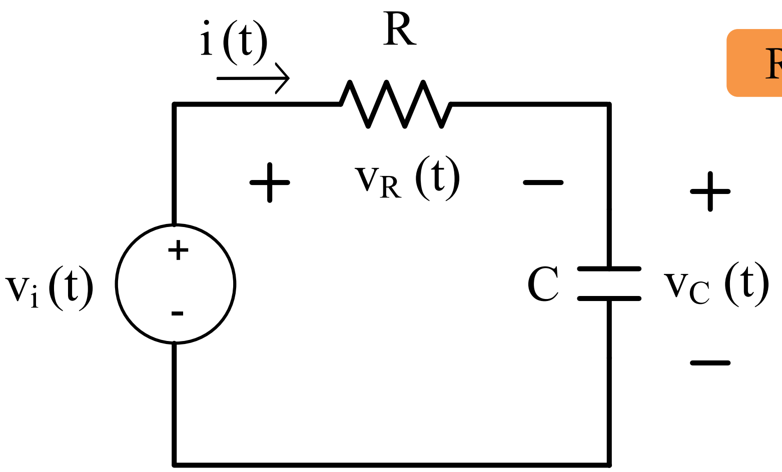


FIRST-ORDER CIRCUITS

Analysis in time- and s-domain

FIRST-ORDER CIRCUITS

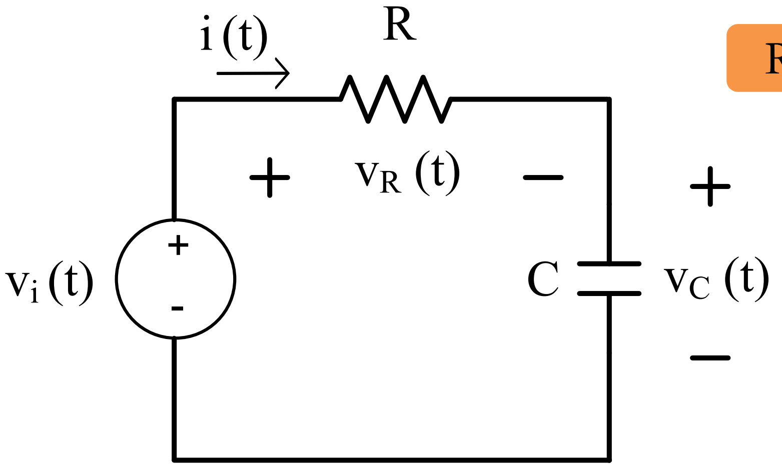
Analysis in time- and s-domain



RC circuit

FIRST-ORDER CIRCUITS

Analysis in time- and s-domain



RC circuit

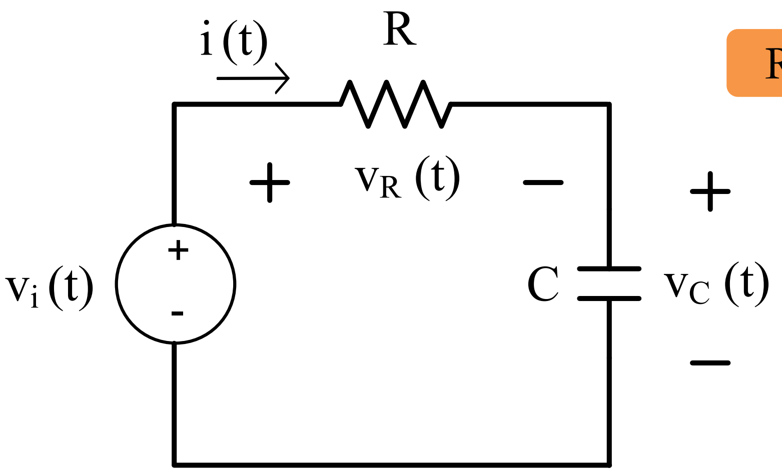
$$v_i(t) = v_R(t) + v_C(t)$$

$$\frac{dv_C(t)}{dt} + \frac{v_C(t)}{RC} = \frac{v_i(t)}{RC}$$

$$v_i(t) = Ri(t) + \frac{1}{C}\int i(t)dt \rightarrow \frac{di(t)}{dt} + \frac{i(t)}{RC} = \frac{1}{R}\frac{dv_i(t)}{dt}$$

FIRST-ORDER CIRCUITS

Analysis in time- and s-domain



RC circuit

$$v_i(t) = v_R(t) + v_C(t)$$

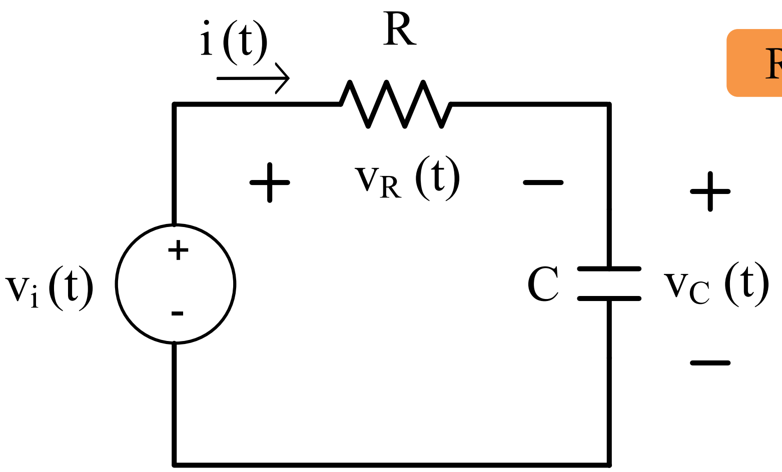
$$\frac{dv_C(t)}{dt} + \frac{v_C(t)}{RC} = \frac{v_i(t)}{RC}$$

$$v_i(t) = Ri(t) + \frac{1}{C}\int i(t)dt \rightarrow \frac{di(t)}{dt} + \frac{i(t)}{RC} = \frac{1}{R}\frac{dv_i(t)}{dt}$$

Concept of transfer function

FIRST-ORDER CIRCUITS

Analysis in time- and s-domain



RC circuit

$$v_{\mathrm{i}}(t) = v_{\mathrm{R}}(t) + v_{\mathrm{C}}(t)$$

$$\frac{\mathrm{d}v_{\mathrm{C}}(t)}{\mathrm{d}t} + \frac{v_{\mathrm{C}}(t)}{RC} = \frac{v_{\mathrm{i}}(t)}{RC}$$

$$v_{\mathrm{i}}(t) = Ri(t) + \frac{1}{C}\int i(t)\mathrm{d}t \rightarrow \frac{\mathrm{d}i(t)}{\mathrm{d}t} + \frac{i(t)}{RC} = \frac{1}{R}\frac{\mathrm{d}v_{\mathrm{i}}(t)}{\mathrm{d}t}$$

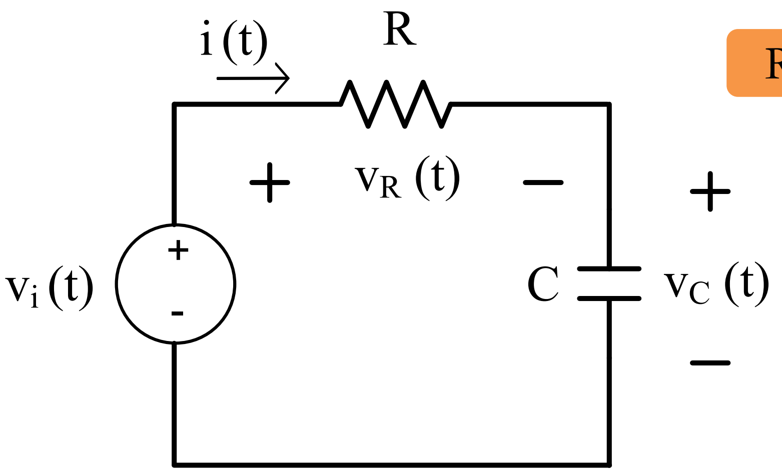
$$V_{\mathrm{C}}(s)\left[s + \frac{1}{RC}\right] = \frac{V_{\mathrm{i}}(s)}{RC} \rightarrow \frac{V_{\mathrm{C}}(s)}{V_{\mathrm{i}}(s)} = \frac{\frac{1}{RC}}{s + \frac{1}{RC}}$$

Concept of transfer function

$$I(s)\left[s + \frac{1}{RC}\right] = \frac{V_{\mathrm{i}}(s)}{RC} \rightarrow \frac{I(s)}{V_{\mathrm{i}}(s)} = \frac{\frac{1}{R}s}{s + \frac{1}{RC}}$$

FIRST-ORDER CIRCUITS

Analysis in time- and s-domain



RC circuit

$$v_i(t) = v_R(t) + v_C(t)$$

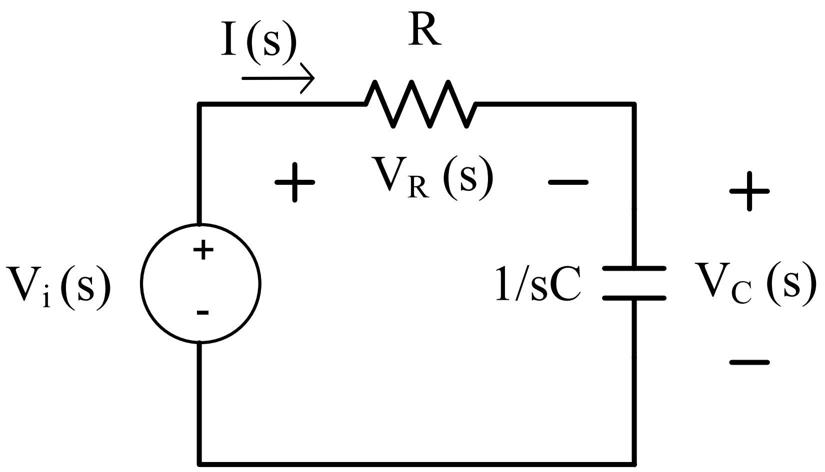
$$\frac{dv_C(t)}{dt} + \frac{v_C(t)}{RC} = \frac{v_i(t)}{RC}$$

$$v_i(t) = Ri(t) + \frac{1}{C}\int i(t)dt \rightarrow \frac{di(t)}{dt} + \frac{i(t)}{RC} = \frac{1}{R}\frac{dv_i(t)}{dt}$$

Concept of transfer function

$$V_C(s)\left[s + \frac{1}{RC}\right] = \frac{V_i(s)}{RC} \rightarrow \frac{V_C(s)}{V_i(s)} = \frac{\frac{1}{RC}}{s + \frac{1}{RC}}$$

$$I(s)\left[s + \frac{1}{RC}\right] = \frac{V_i(s)}{RC} \rightarrow \frac{I(s)}{V_i(s)} = \frac{\frac{1}{R}s}{s + \frac{1}{RC}}$$

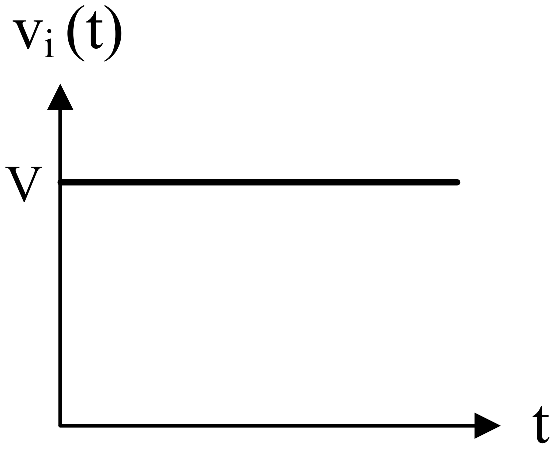


FIRST-ORDER CIRCUITS

Transient and steady-state response

FIRST-ORDER CIRCUITS

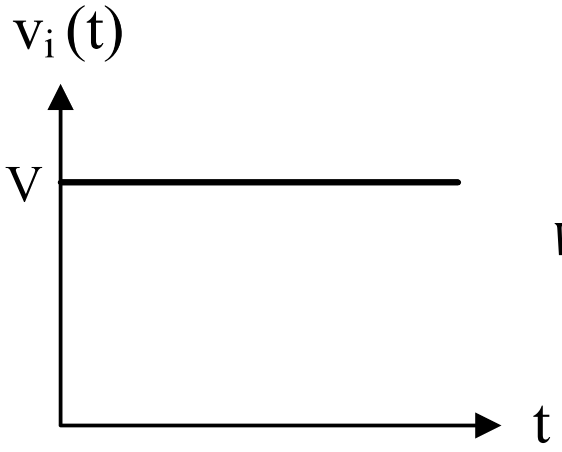
Transient and steady-state response



$$v_i(t) = Vu(t) \rightarrow V_i(s) = \frac{V}{s}$$

FIRST-ORDER CIRCUITS

Transient and steady-state response

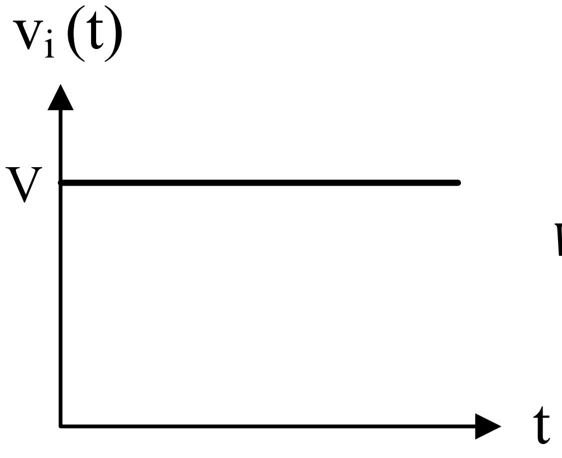


$$v_{\mathrm{i}}(t) = Vu(t) \rightarrow V_{\mathrm{i}}(s) = \frac{V}{s}$$

$$V_{\mathrm{C}}(s) = \frac{V}{s}\frac{\frac{1}{RC}}{s + \frac{1}{RC}} \rightarrow v_{\mathrm{C}}(t) = V\left(1 - e^{-t/\tau}\right)$$

FIRST-ORDER CIRCUITS

Transient and steady-state response



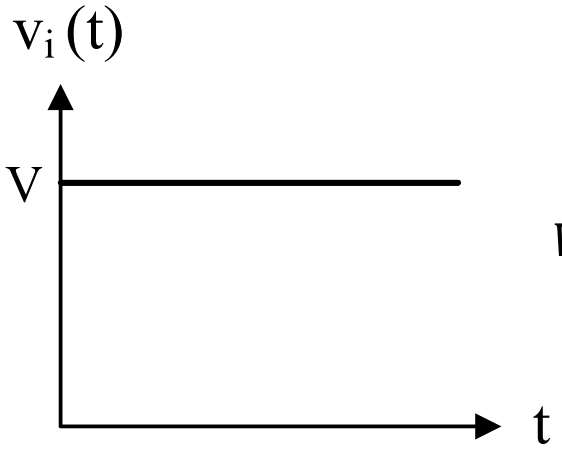
$$v_{\mathrm{i}}(t) = Vu(t) \rightarrow V_{\mathrm{i}}(s) = \frac{V}{s}$$

$$V_{\mathrm{C}}(s) = \frac{V}{s}\frac{\frac{1}{RC}}{s + \frac{1}{RC}} \rightarrow v_{\mathrm{C}}(t) = V\left(1 - e^{-t/\tau}\right)$$

Time-constant $\tau = RC$

FIRST-ORDER CIRCUITS

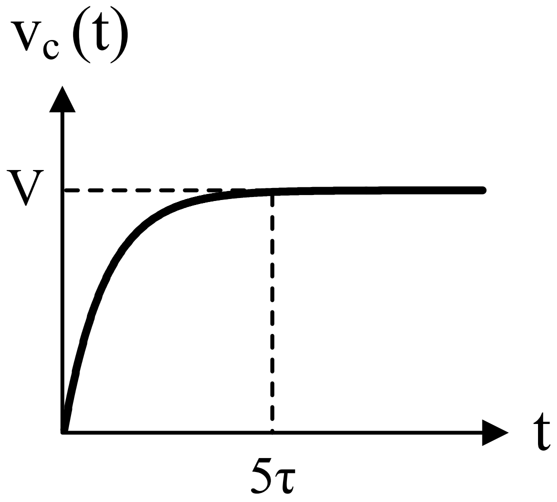
Transient and steady-state response



$$v_{\mathrm{i}}(t) = Vu(t) \rightarrow V_{\mathrm{i}}(s) = \frac{V}{s}$$

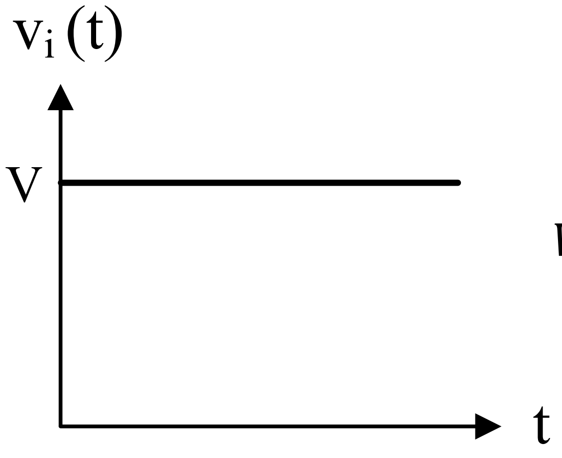
$$V_{\mathrm{C}}(s) = \frac{V}{s}\frac{\frac{1}{RC}}{s + \frac{1}{RC}} \rightarrow v_{\mathrm{C}}(t) = V\left(1 - e^{-t/\tau}\right)$$

Time-constant $\tau = RC$



FIRST-ORDER CIRCUITS

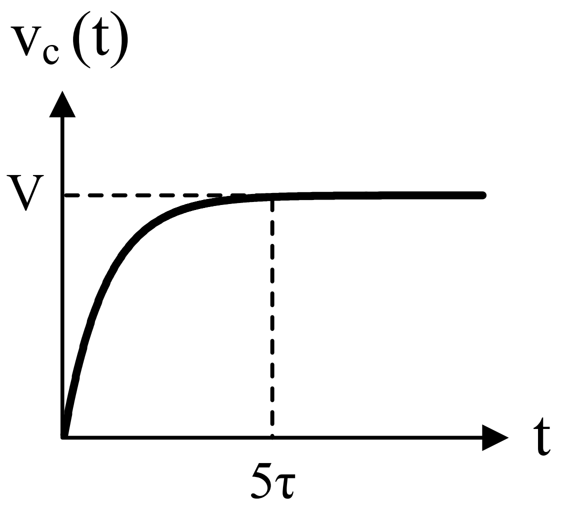
Transient and steady-state response



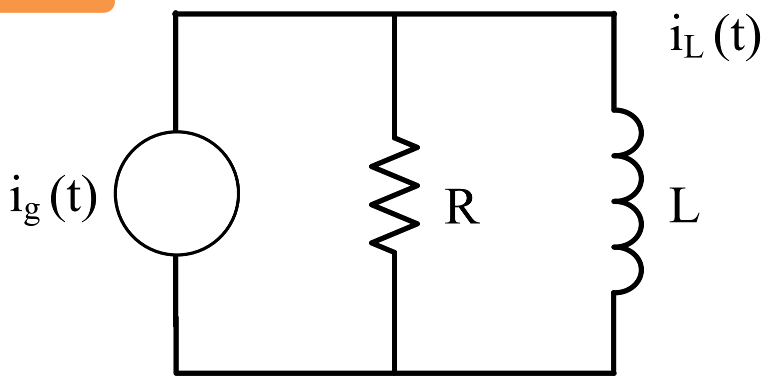
$$v_i(t) = Vu(t) \rightarrow V_i(s) = \frac{V}{s}$$

$$V_C(s) = \frac{V}{s}\frac{\frac{1}{RC}}{s+\frac{1}{RC}} \rightarrow v_C(t) = V\left(1 - e^{-t/\tau}\right)$$

Time-constant $\tau = RC$

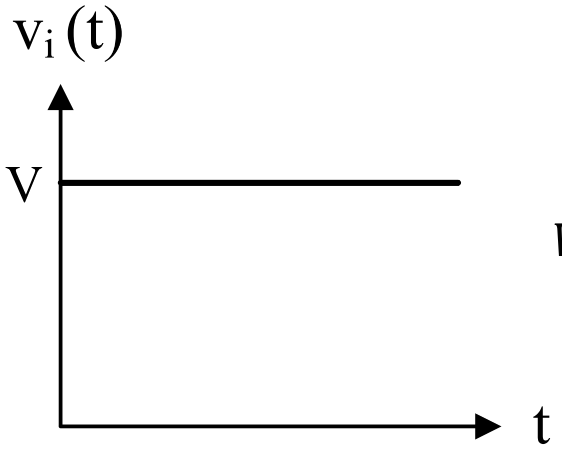


RL circuit



FIRST-ORDER CIRCUITS

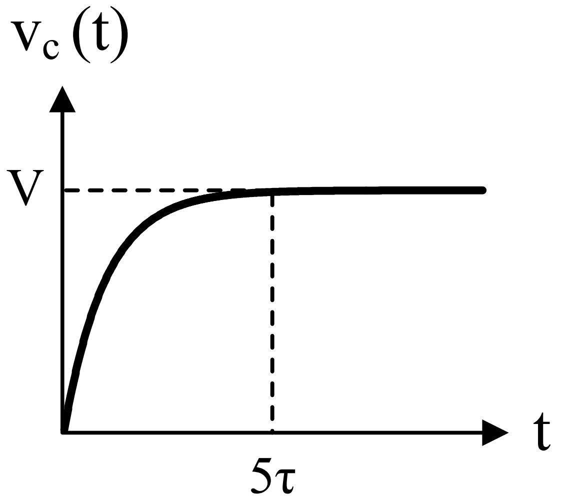
Transient and steady-state response



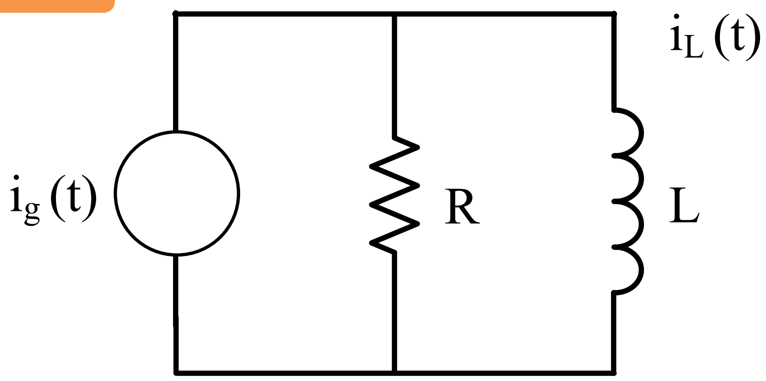
$$v_i(t) = Vu(t) \rightarrow V_i(s) = \frac{V}{s}$$

$$V_C(s) = \frac{V}{s} \frac{\frac{1}{RC}}{s + \frac{1}{RC}} \rightarrow v_C(t) = V\left(1 - e^{-t/\tau}\right)$$

Time-constant $\tau = RC$



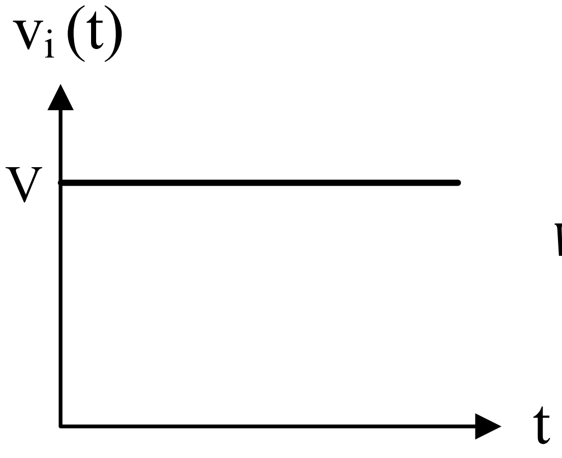
RL circuit



$$i_g(t) = Iu(t)$$

FIRST-ORDER CIRCUITS

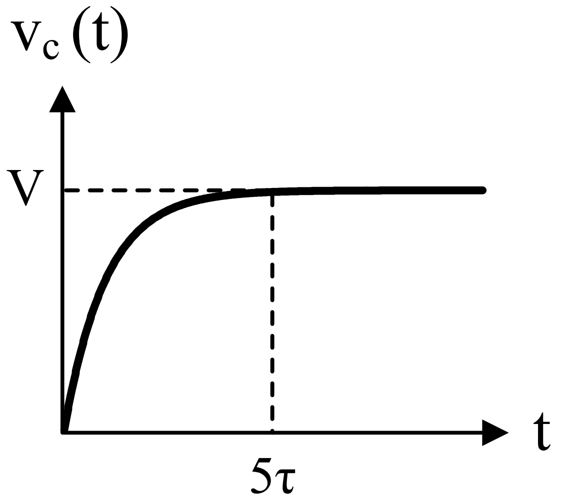
Transient and steady-state response



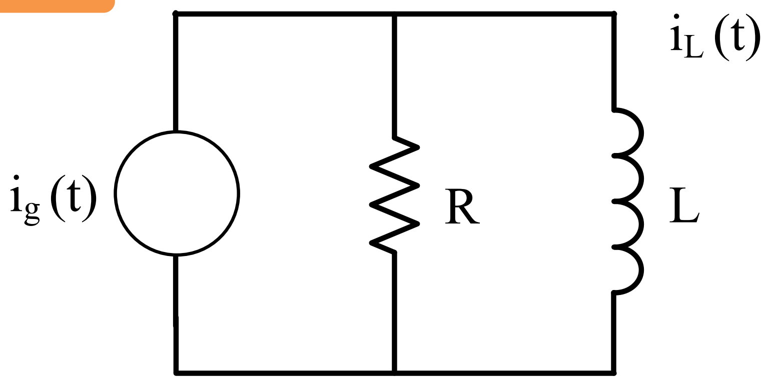
$$v_i(t) = Vu(t) \rightarrow V_i(s) = \frac{V}{s}$$

$$V_C(s) = \frac{V}{s} \frac{\frac{1}{RC}}{s + \frac{1}{RC}} \rightarrow v_C(t) = V\left(1 - e^{-t/\tau}\right)$$

Time-constant $\tau = RC$



RL circuit



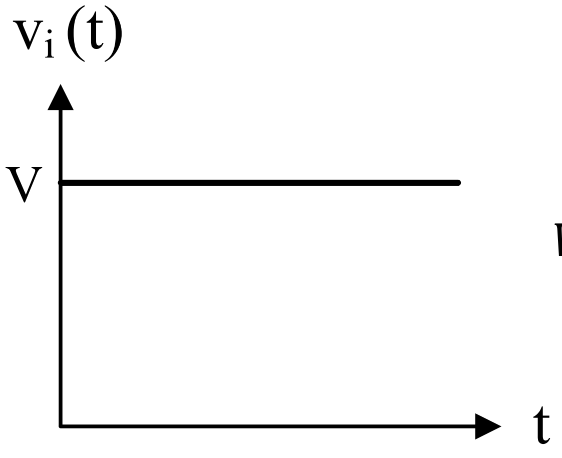
$$i_g(t) = Iu(t)$$

$$i_L(t) = I\left(1 - e^{-t/\tau}\right)$$

$$\tau = \frac{L}{R}$$

FIRST-ORDER CIRCUITS

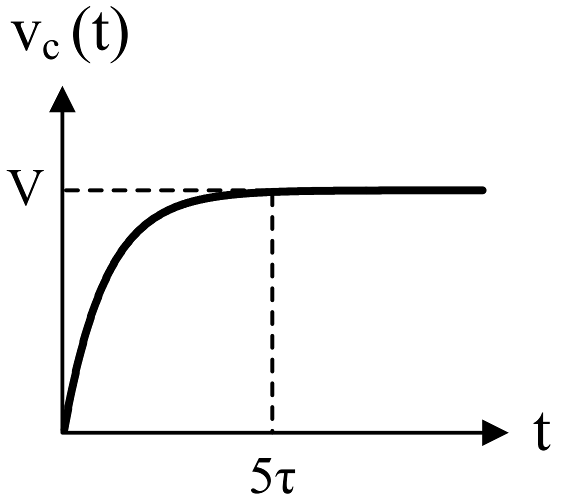
Transient and steady-state response



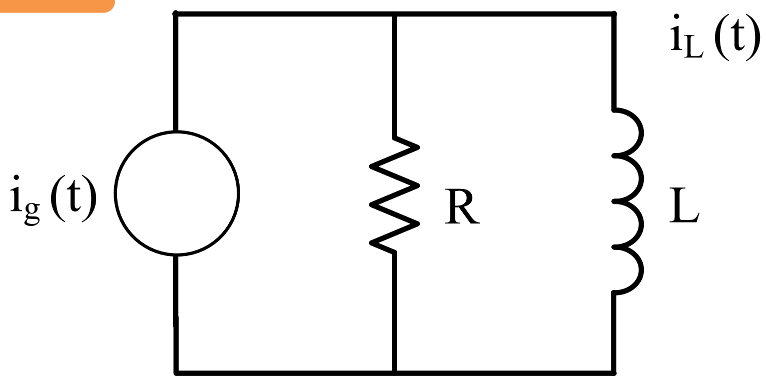
$$v_i(t) = Vu(t) \rightarrow V_i(s) = \frac{V}{s}$$

$$V_C(s) = \frac{V}{s}\frac{\frac{1}{RC}}{s+\frac{1}{RC}} \rightarrow v_C(t) = V\left(1 - e^{-t/\tau}\right)$$

Time-constant $\tau = RC$



RL circuit



$$i_g(t) = Iu(t)$$

$$i_L(t) = I\left(1 - e^{-t/\tau}\right)$$

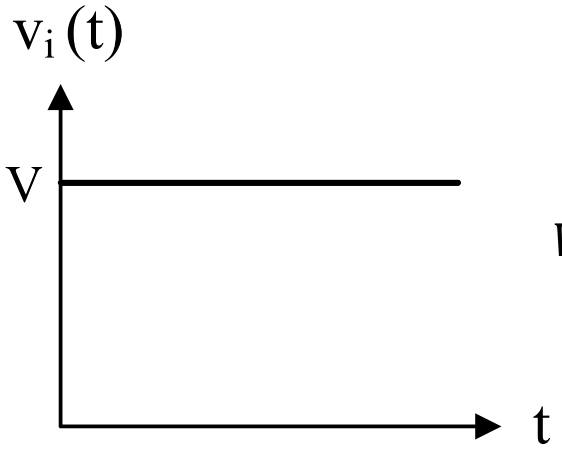
$$\tau = \frac{L}{R}$$

- **General theory:**

$$x(t) = x(\infty) + [x(0) - x(\infty)]e^{-\frac{t}{\tau}}$$

FIRST-ORDER CIRCUITS

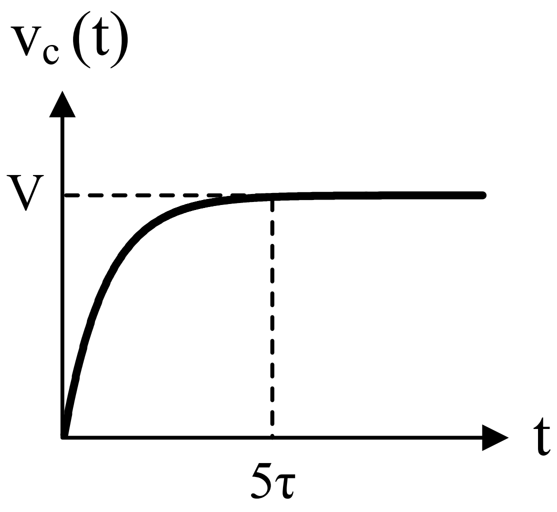
Transient and steady-state response



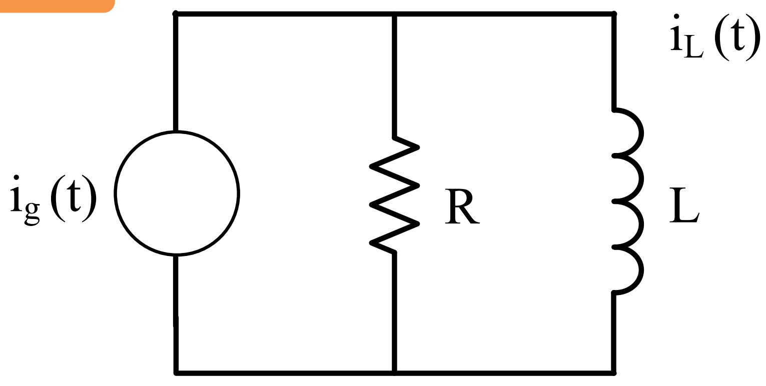
$$v_\mathrm{i}(t) = Vu(t) \rightarrow V_\mathrm{i}(s) = \frac{V}{s}$$

$$V_\mathrm{C}(s) = \frac{V}{s}\frac{\frac{1}{RC}}{s + \frac{1}{RC}} \rightarrow v_\mathrm{C}(t) = V\left(1 - e^{-t/\tau}\right)$$

Time-constant $\tau = RC$



RL circuit



$$i_\mathrm{g}(t) = Iu(t)$$

$$i_\mathrm{L}(t) = I\left(1 - e^{-t/\tau}\right)$$

$$\tau = \frac{L}{R}$$

- **General theory:**

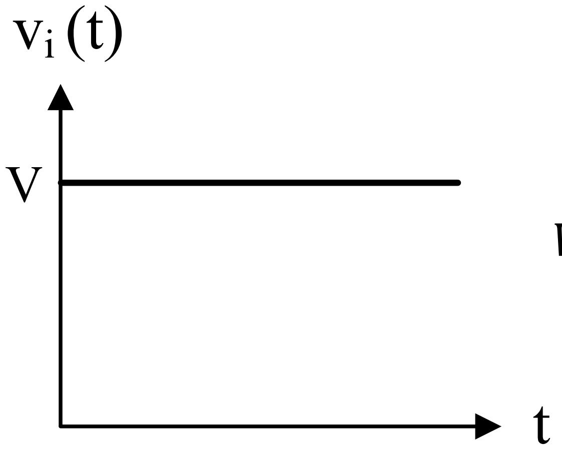
$$x(t) = x(\infty) + [x(0) - x(\infty)]e^{-\frac{t}{\tau}}$$

➢ *Transient response:* $x_\mathrm{t}(t) = [x(0) - x(\infty)]e^{-\frac{t}{\tau}}$

"momentary event"

FIRST-ORDER CIRCUITS

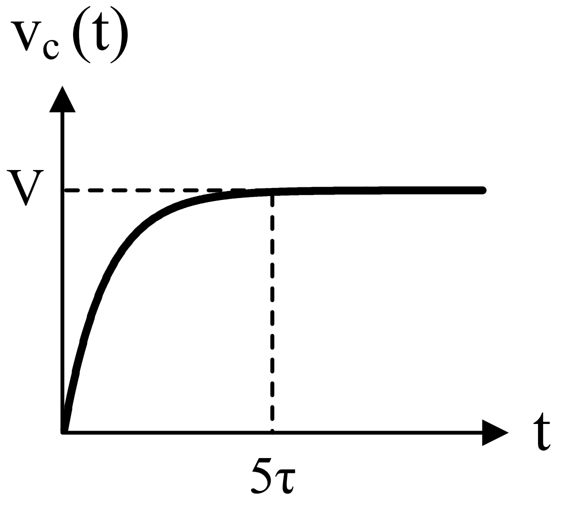
Transient and steady-state response



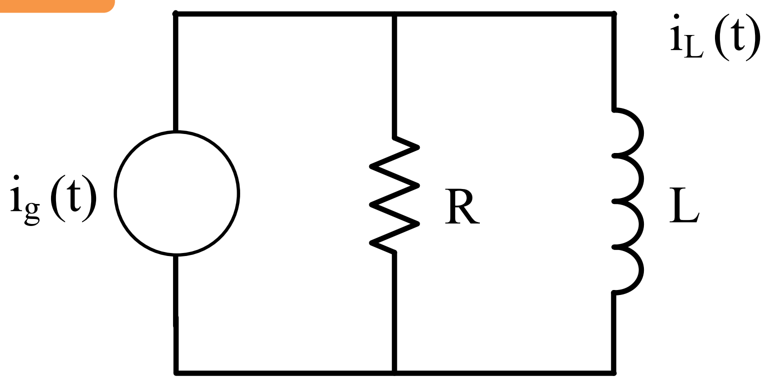
$$v_i(t) = Vu(t) \rightarrow V_i(s) = \frac{V}{s}$$

$$V_C(s) = \frac{V}{s}\frac{\frac{1}{RC}}{s + \frac{1}{RC}} \rightarrow v_C(t) = V\left(1 - e^{-t/\tau}\right)$$

Time-constant $\tau = RC$



RL circuit



$$i_g(t) = Iu(t)$$

$$i_L(t) = I\left(1 - e^{-t/\tau}\right)$$

$$\tau = \frac{L}{R}$$

- **General theory:**

$$x(t) = x(\infty) + [x(0) - x(\infty)]e^{-\frac{t}{\tau}}$$

- *Transient response:* $x_t(t) = [x(0) - x(\infty)]e^{-\frac{t}{\tau}}$

  "momentary event"

- *Steady-state response:* $x_{ss}(t) = x(\infty)$

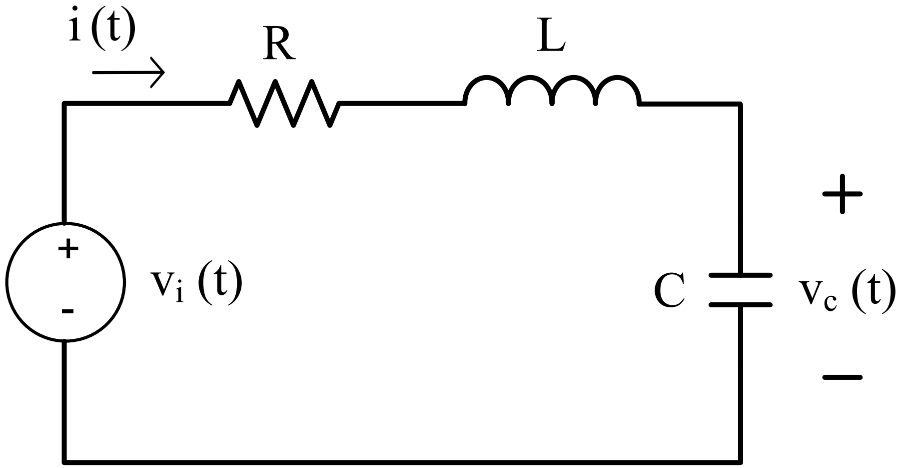
  "exists a long time after the switching"

SECOND-ORDER CIRCUITS

RLC circuit

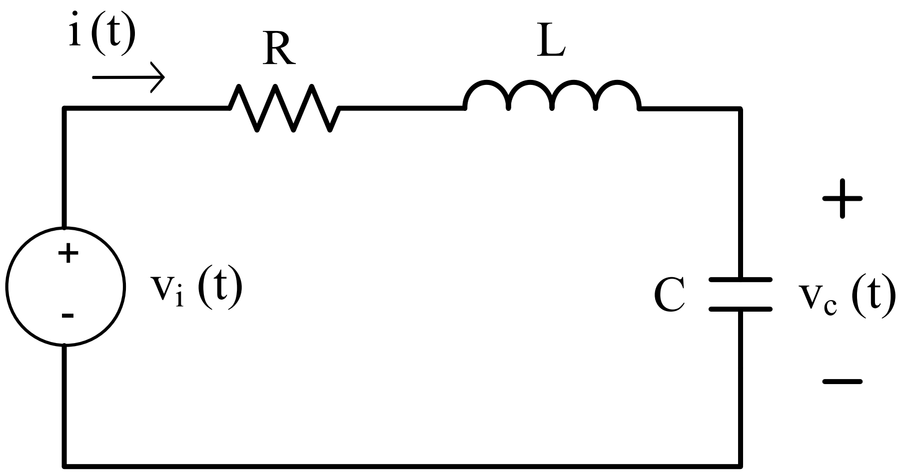
SECOND-ORDER CIRCUITS

RLC circuit



SECOND-ORDER CIRCUITS

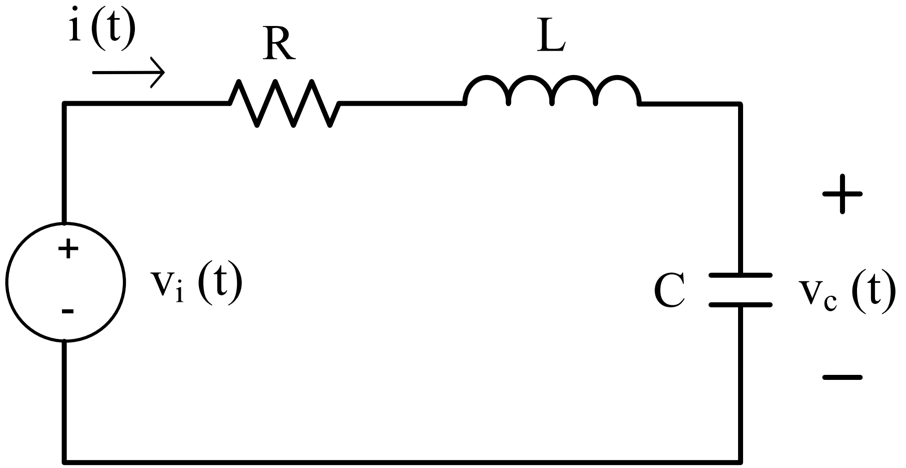
RLC circuit



$$\frac{V_C(s)}{V_i(s)} = \frac{\frac{1}{LC}}{s^2 + \frac{R}{L}s + \frac{1}{LC}}$$

SECOND-ORDER CIRCUITS

RLC circuit

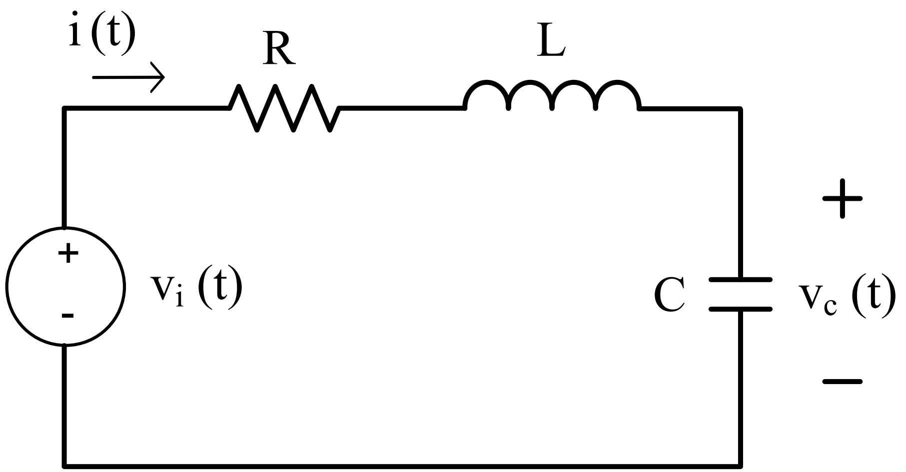


$$\frac{V_C(s)}{V_i(s)} = \frac{\frac{1}{LC}}{s^2 + \frac{R}{L}s + \frac{1}{LC}}$$

$$s^2 + 2\xi\omega_n s + \omega_n^2 \rightarrow \begin{cases} 2\xi\omega_n = \frac{R}{L}; \quad \xi = \frac{R}{2}\sqrt{\frac{C}{L}} \\ \omega_n^2 = \frac{1}{LC}; \quad \omega_n = \frac{1}{\sqrt{LC}} \end{cases}$$

SECOND-ORDER CIRCUITS

RLC circuit



$$\frac{V_C(s)}{V_i(s)} = \frac{\frac{1}{LC}}{s^2 + \frac{R}{L}s + \frac{1}{LC}}$$

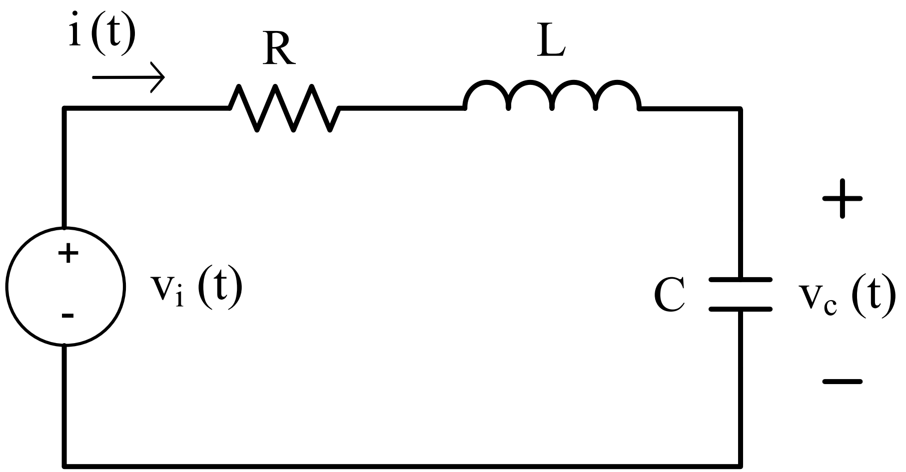
$$s^2 + 2\xi\omega_n s + \omega_n^2 \rightarrow \begin{cases} 2\xi\omega_n = \frac{R}{L}; \quad \xi = \frac{R}{2}\sqrt{\frac{C}{L}} \\ \omega_n^2 = \frac{1}{LC}; \quad \omega_n = \frac{1}{\sqrt{LC}} \end{cases}$$

Damping factor

Undamped radian frequency

SECOND-ORDER CIRCUITS

RLC circuit



$$v_{\mathrm{i}}(t) = Vu(t) \rightarrow V_{\mathrm{i}}(s) = \frac{V}{s}$$

$$\frac{V_{\mathrm{C}}(s)}{V_{\mathrm{i}}(s)} = \frac{\frac{1}{LC}}{s^2 + \frac{R}{L}s + \frac{1}{LC}}$$

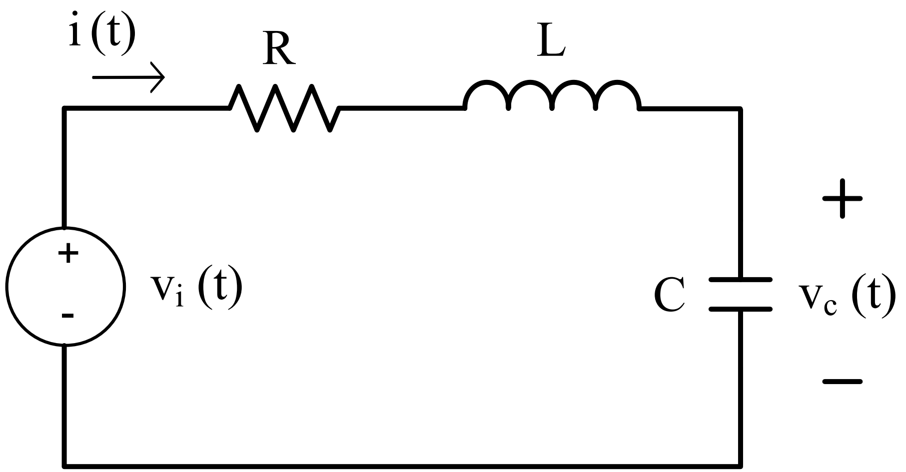
$$s^2 + 2\xi\omega_{\mathrm{n}}s + \omega_{\mathrm{n}}^2 \rightarrow \begin{cases} 2\xi\omega_{\mathrm{n}} = \frac{R}{L}; \quad \xi = \frac{R}{2}\sqrt{\frac{C}{L}} \\ \omega_{\mathrm{n}}^2 = \frac{1}{LC}; \quad \omega_{\mathrm{n}} = \frac{1}{\sqrt{LC}} \end{cases}$$

Damping factor

Undamped radian frequency

SECOND-ORDER CIRCUITS

RLC circuit



$$v_{\mathrm{i}}(t) = Vu(t) \rightarrow V_{\mathrm{i}}(s) = \frac{V}{s}$$

$$v_{\mathrm{c}}(t) = v_{\mathrm{c,t}}(t) + v_{\mathrm{c,ss}}(t)$$

$$\frac{V_{\mathrm{C}}(s)}{V_{\mathrm{i}}(s)} = \frac{\frac{1}{LC}}{s^2 + \frac{R}{L}s + \frac{1}{LC}}$$

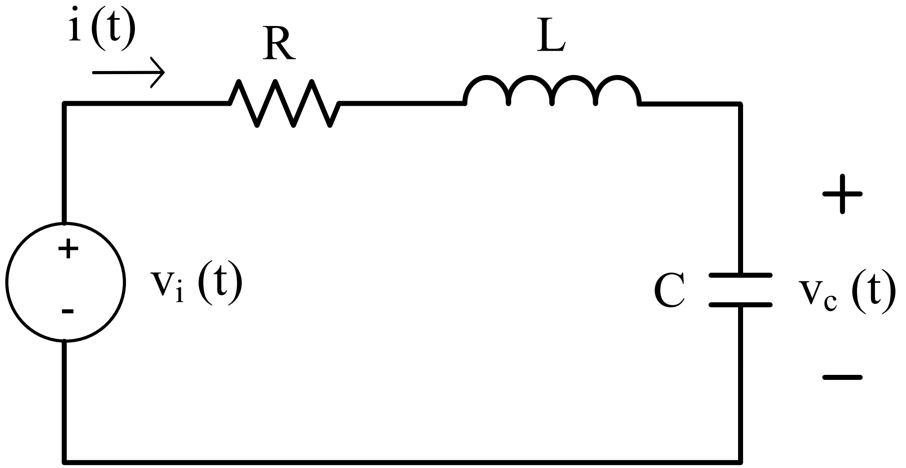
$$s^2 + 2\xi\omega_{\mathrm{n}}s + \omega_{\mathrm{n}}^2 \rightarrow \begin{cases} 2\xi\omega_{\mathrm{n}} = \frac{R}{L}; \quad \xi = \frac{R}{2}\sqrt{\frac{C}{L}} \\ \omega_{\mathrm{n}}^2 = \frac{1}{LC}; \quad \omega_{\mathrm{n}} = \frac{1}{\sqrt{LC}} \end{cases}$$

Damping factor

Undamped radian frequency

SECOND-ORDER CIRCUITS

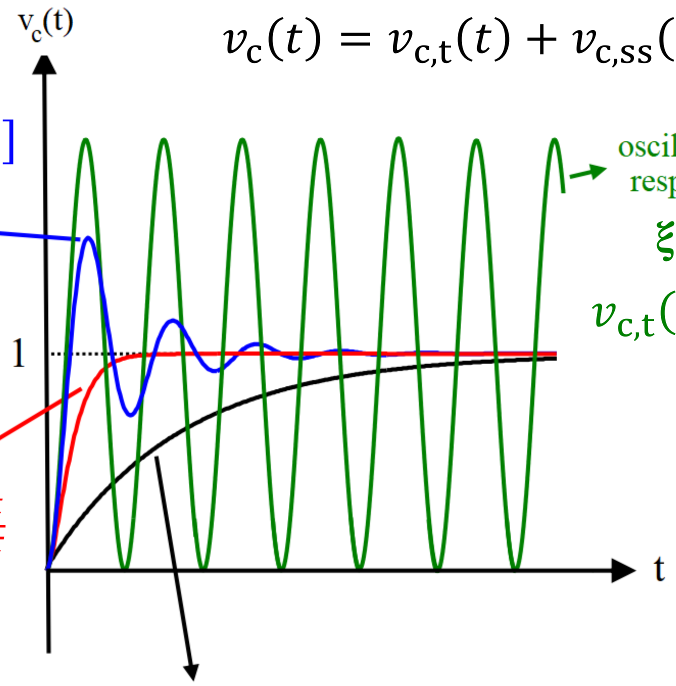
RLC circuit



$$v_i(t) = Vu(t) \rightarrow V_i(s) = \frac{V}{s}$$

$$v_c(t) = v_{c,t}(t) + v_{c,ss}(t)$$

$$\frac{V_C(s)}{V_i(s)} = \frac{\frac{1}{LC}}{s^2 + \frac{R}{L}s + \frac{1}{LC}}$$



underdamped response

0<ξ<1

$$v_{c,t}(t) = e^{-\frac{t}{\tau}}[K_1\cos(\omega_d t) + K_2\sin(\omega_d t)]$$

oscillatory response

ξ=0

$$v_{c,t}(t) = K\cos(\omega_n t)$$

ξ=1

critically damped

$$v_{c,t}(t) = K_1 t e^{-\frac{t}{\tau}} + K_2 e^{-\frac{t}{\tau}}$$

overdamped response

ξ>1

$$v_{c,t}(t) = K_1 e^{-\frac{t}{\tau_1}} + K_2 e^{-\frac{t}{\tau_2}}$$

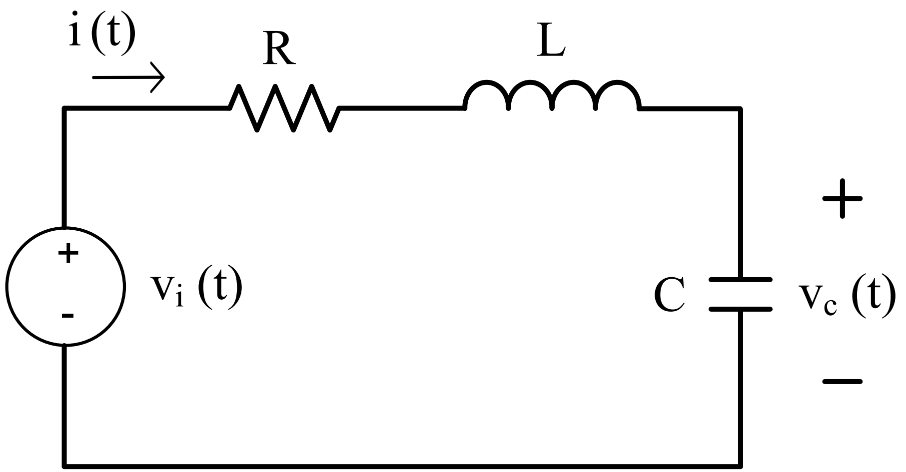
$$s^2 + 2\xi\omega_n s + \omega_n^2 \rightarrow \begin{cases} 2\xi\omega_n = \frac{R}{L}; \quad \xi = \frac{R}{2}\sqrt{\frac{C}{L}} \\ \omega_n^2 = \frac{1}{LC}; \quad \omega_n = \frac{1}{\sqrt{LC}} \end{cases}$$

Damping factor

Undamped radian frequency

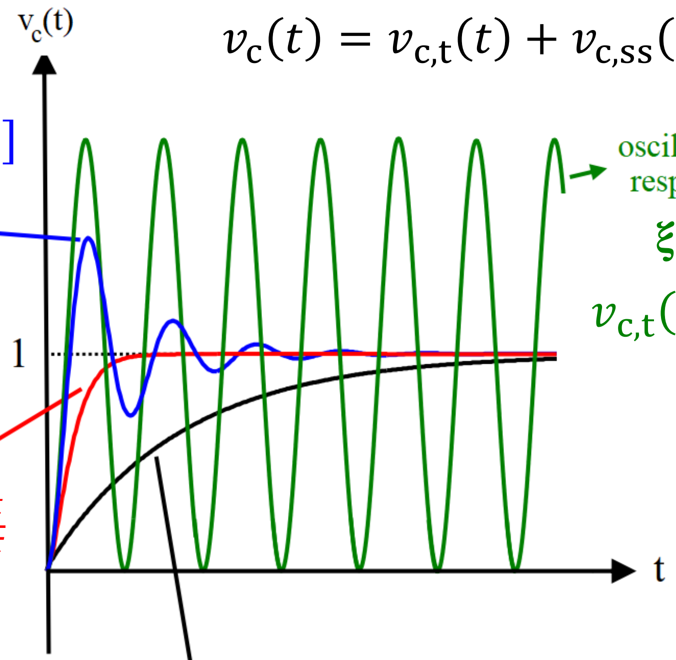
SECOND-ORDER CIRCUITS

RLC circuit



$$v_{\mathrm{i}}(t) = Vu(t) \rightarrow V_{\mathrm{i}}(s) = \frac{V}{s}$$

$$v_{\mathrm{c}}(t) = v_{\mathrm{c,t}}(t) + v_{\mathrm{c,ss}}(t)$$



oscillatory response

ξ=0

$$v_{\mathrm{c,t}}(t) = K\cos(\omega_{\mathrm{n}} t)$$

underdamped response

0<ξ<1

$$v_{\mathrm{c,t}}(t) = e^{-\frac{t}{\tau}}[K_1\cos(\omega_{\mathrm{d}} t) + K_2\sin(\omega_{\mathrm{d}} t)]$$

ξ=1

critically damped

$$v_{\mathrm{c,t}}(t) = K_1 t e^{-\frac{t}{\tau}} + K_2 e^{-\frac{t}{\tau}}$$

overdamped response

ξ>1

$$v_{\mathrm{c,t}}(t) = K_1 e^{-\frac{t}{\tau_1}} + K_2 e^{-\frac{t}{\tau_2}}$$

$$v_{\mathrm{ss}}(t) = V$$

$$\frac{V_{\mathrm{C}}(s)}{V_{\mathrm{i}}(s)} = \frac{\frac{1}{LC}}{s^2 + \frac{R}{L}s + \frac{1}{LC}}$$

$$s^2 + 2\xi\omega_{\mathrm{n}} s + \omega_{\mathrm{n}}^2 \rightarrow \begin{cases} 2\xi\omega_{\mathrm{n}} = \frac{R}{L}; \quad \xi = \frac{R}{2}\sqrt{\frac{C}{L}} \\ \omega_{\mathrm{n}}^2 = \frac{1}{LC}; \quad \omega_{\mathrm{n}} = \frac{1}{\sqrt{LC}} \end{cases}$$

Damping factor

Undamped radian frequency

PRACTICAL EXAMPLES

RLC circuit

PRACTICAL EXAMPLES

RLC circuit

Defibrillator

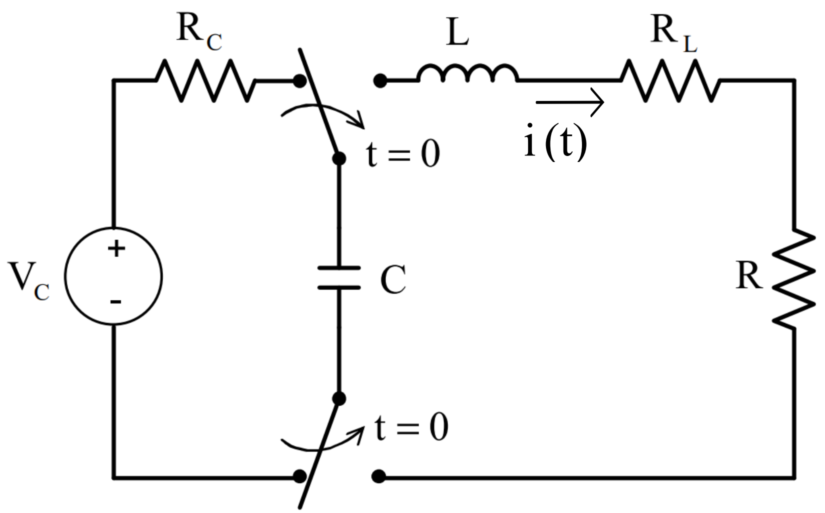
Device that gives a high energy electric shock to the heart of someone who is in cardiac arrest.

PRACTICAL EXAMPLES

RLC circuit

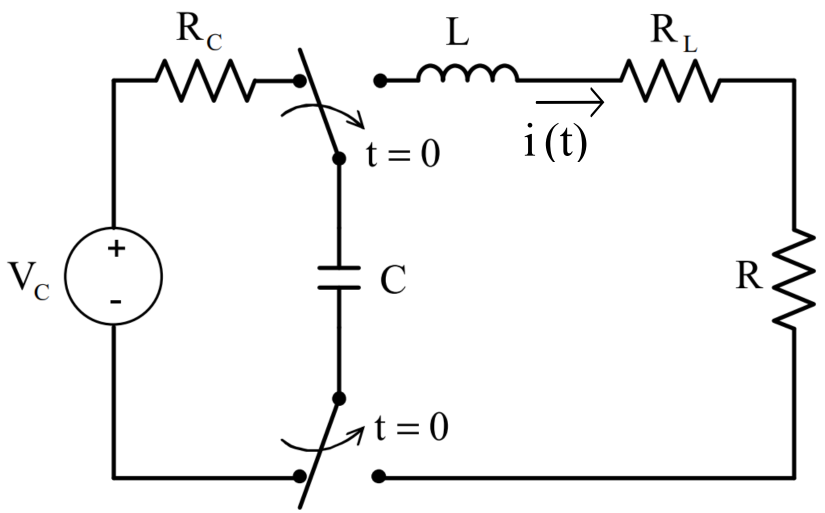
Defibrillator

Device that gives a high energy electric shock to the heart of someone who is in cardiac arrest.



PRACTICAL EXAMPLES

RLC circuit

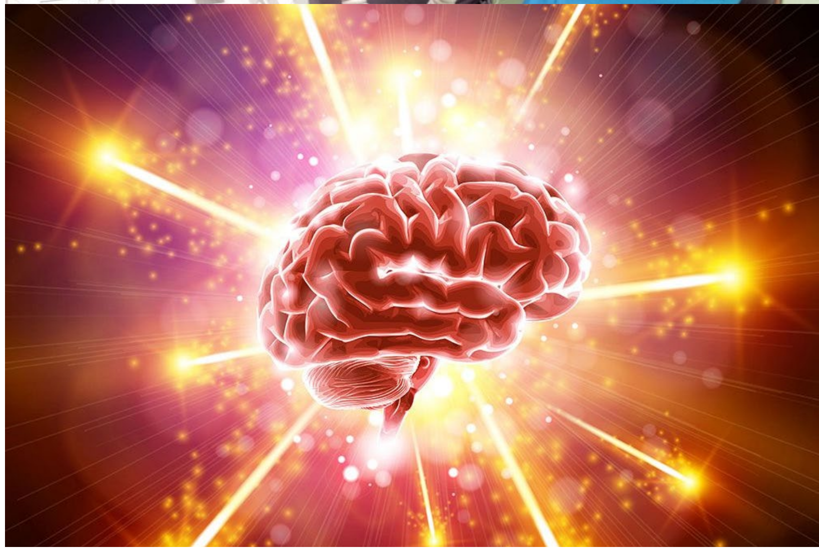


Defibrillator

Device that gives a high energy electric shock to the heart of someone who is in cardiac arrest.

Transcranial magnetic stimulation (TMS)

Noninvasive procedure that uses magnetic fields to stimulate nerve cells in the brain to improve symptoms of depression.

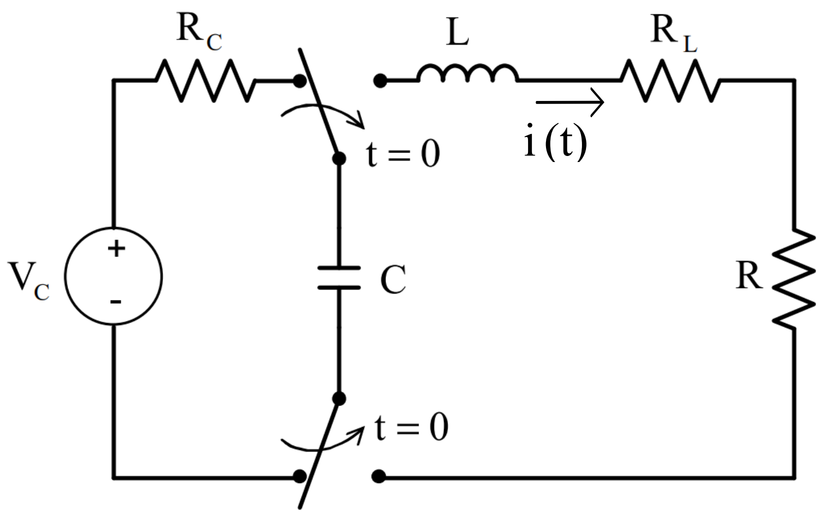


PRACTICAL EXAMPLES

RLC circuit

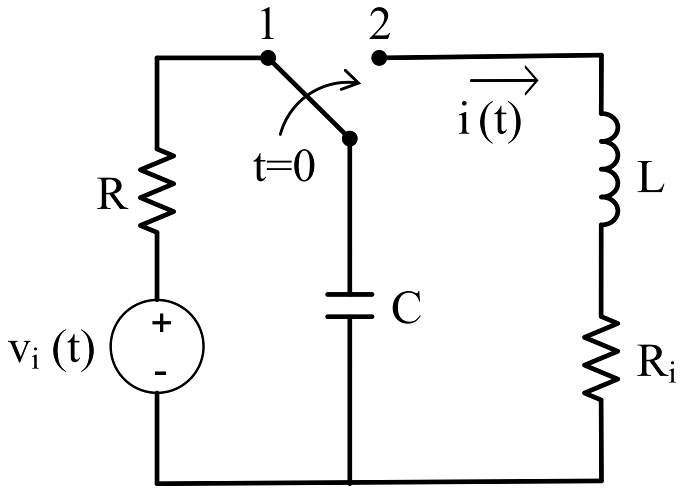
Defibrillator

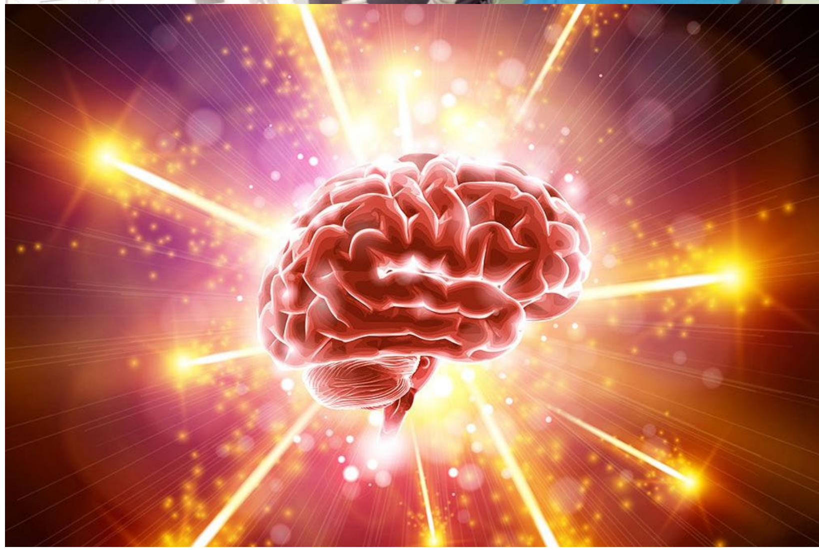
Device that gives a high energy electric shock to the heart of someone who is in cardiac arrest.



Transcranial magnetic stimulation (TMS)

Noninvasive procedure that uses magnetic fields to stimulate nerve cells in the brain to improve symptoms of depression.





FREQUENCY RESPONSE

Nyquist plots

FREQUENCY RESPONSE

Nyquist plots

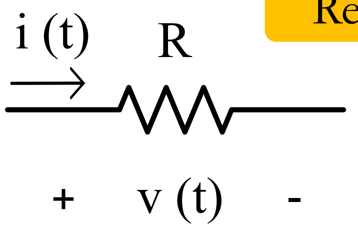
- From s-domain to **physical frequencies**: $s = j\omega$

FREQUENCY RESPONSE

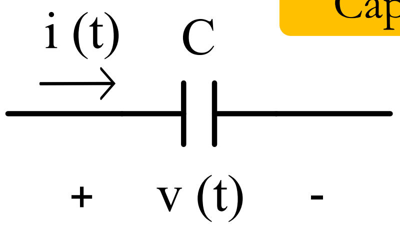
Nyquist plots

➢ From s-domain to **physical frequencies:** $s = j\omega$

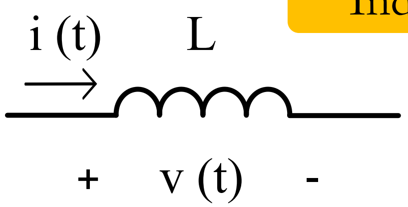
Resistance



Capacitor



Inductor



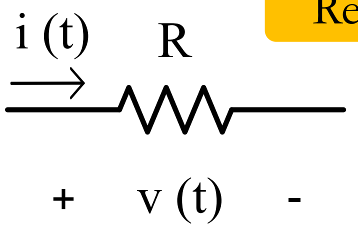
FREQUENCY RESPONSE

Nyquist plots

- From s-domain to **physical frequencies:** $s = \mathrm{j}\omega$

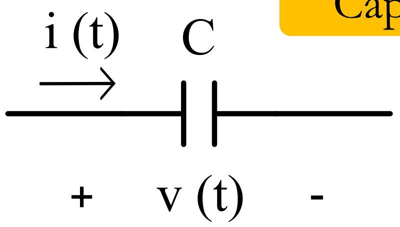
Frequency-domain

Resistance



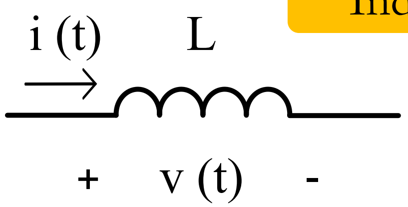
$$Z_{\mathrm{R}}(\omega) = R$$

Capacitor



$$Z_{\mathrm{C}}(\omega) = \frac{1}{\mathrm{j}\omega C}$$

Inductor



$$Z_{\mathrm{L}}(\omega) = \mathrm{j}\omega L$$

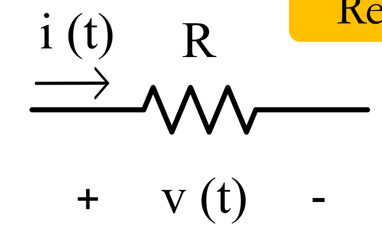
FREQUENCY RESPONSE

Nyquist plots

- From s-domain to **physical frequencies**: $s = j\omega$

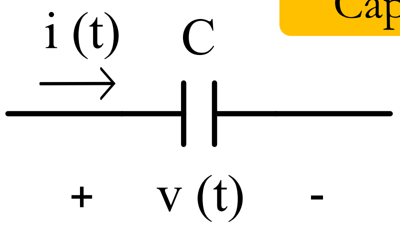
Frequency-domain

Resistance



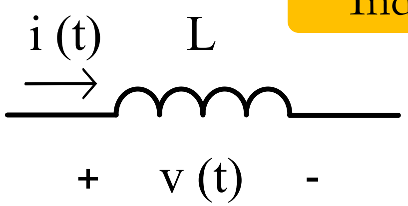
$$Z_R(\omega) = R$$

Capacitor

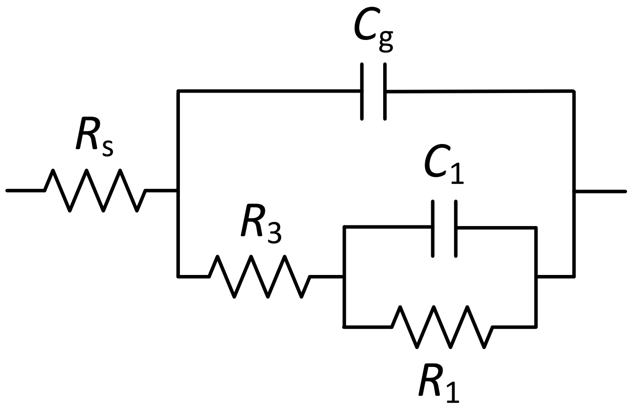


$$Z_C(\omega) = \frac{1}{j\omega C}$$

Inductor



$$Z_L(\omega) = j\omega L$$



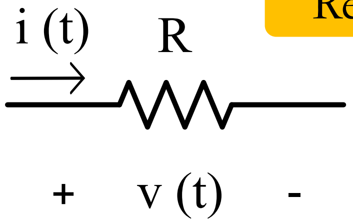
FREQUENCY RESPONSE

Nyquist plots

- From s-domain to **physical frequencies**: $s = j\omega$

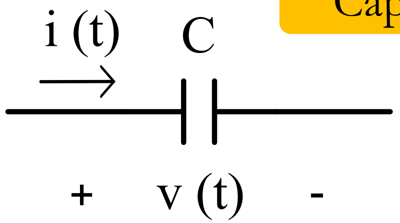
Frequency-domain

Resistance



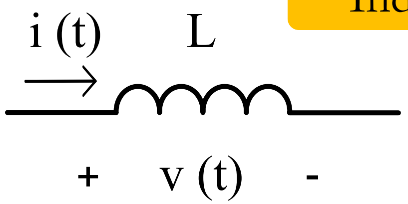
$$Z_R(\omega) = R$$

Capacitor

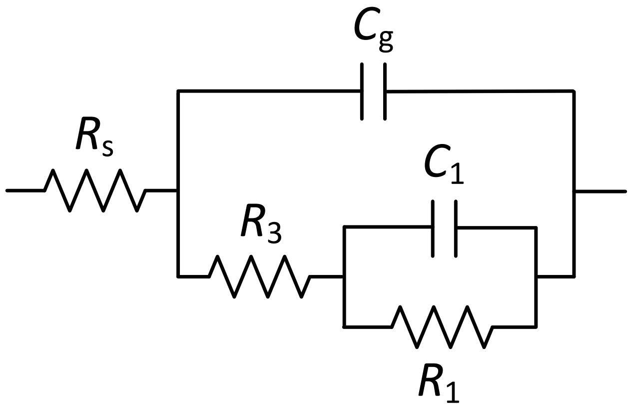


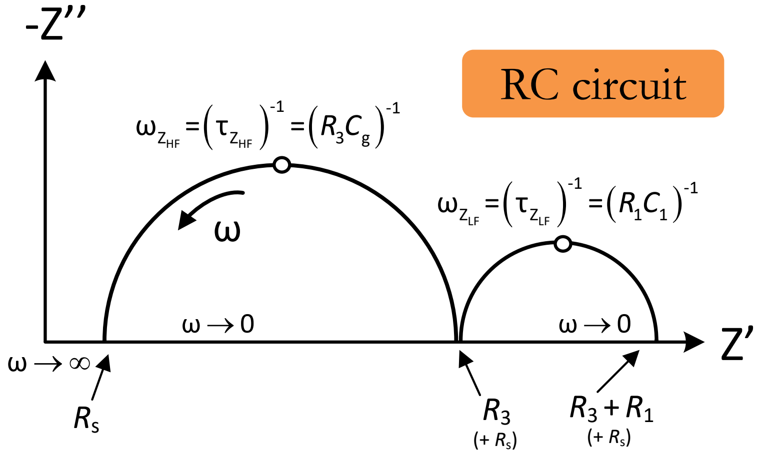
$$Z_C(\omega) = \frac{1}{j\omega C}$$

Inductor



$$Z_L(\omega) = j\omega L$$





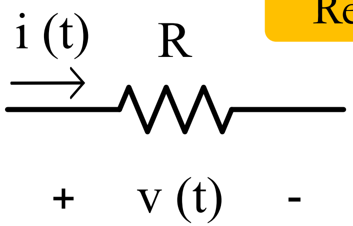
FREQUENCY RESPONSE

Nyquist plots

- From s-domain to **physical frequencies**: $s = \mathrm{j}\omega$

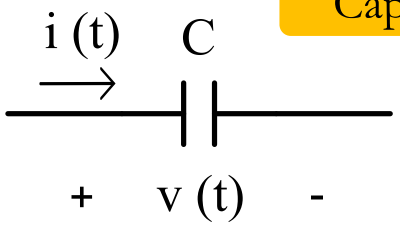
Frequency-domain

Resistance



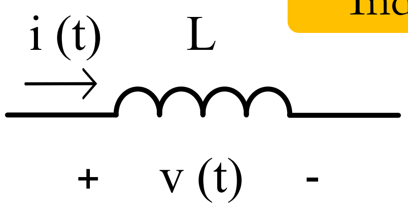
$$Z_R(\omega) = R$$

Capacitor

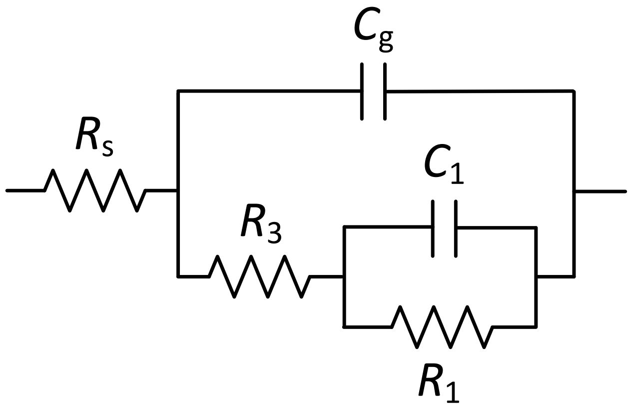


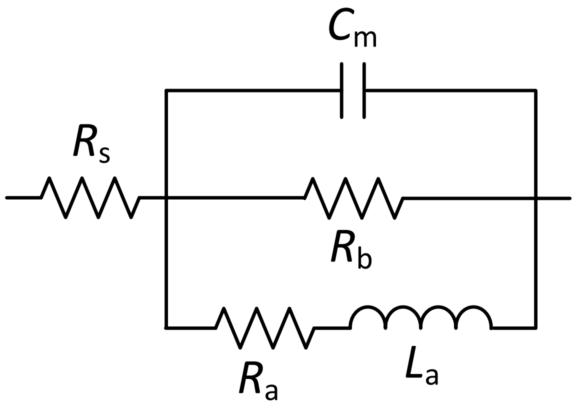
$$Z_C(\omega) = \frac{1}{\mathrm{j}\omega C}$$

Inductor

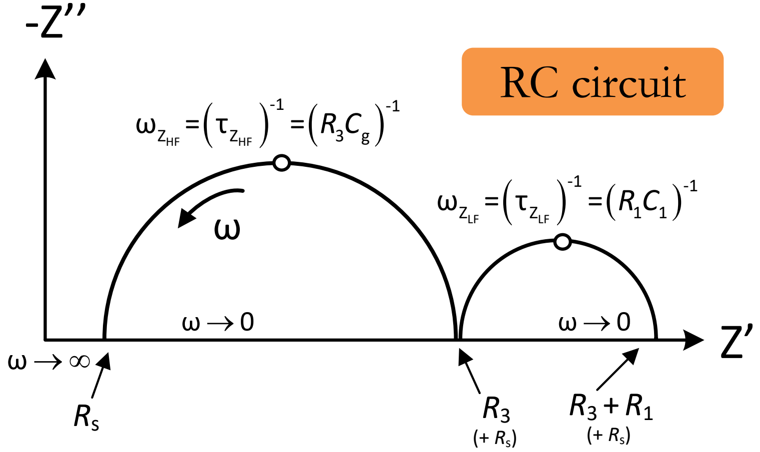


$$Z_L(\omega) = \mathrm{j}\omega L$$





RC circuit



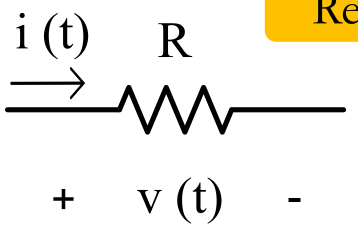
FREQUENCY RESPONSE

Nyquist plots

➢ From s-domain to **physical frequencies**: $s = j\omega$

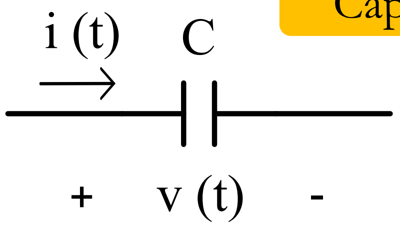
Frequency-domain

Resistance



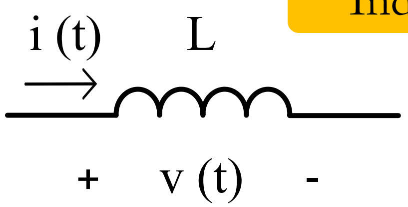
$$Z_R(\omega) = R$$

Capacitor

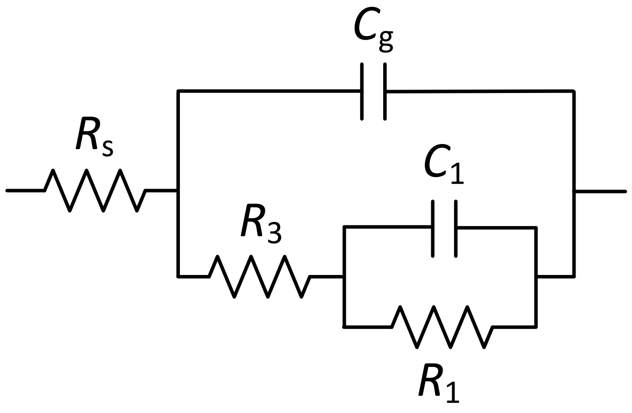


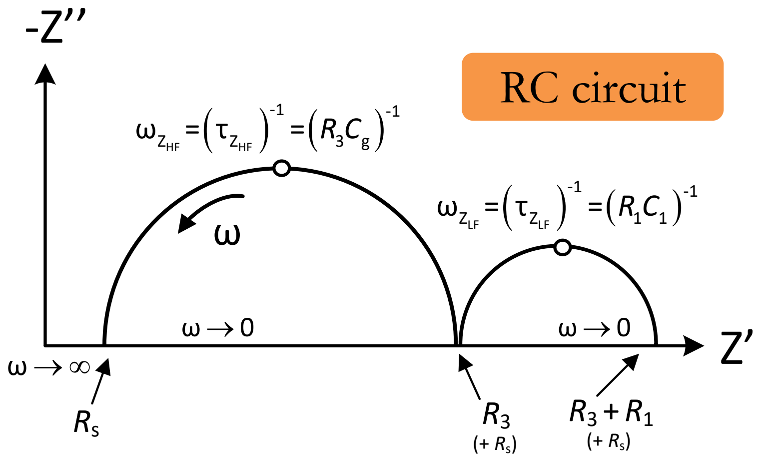
$$Z_C(\omega) = \frac{1}{j\omega C}$$

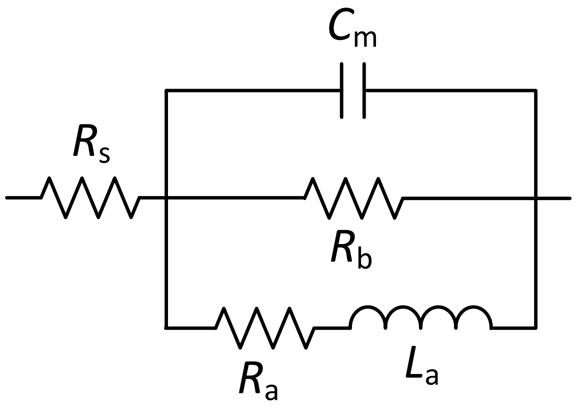
Inductor

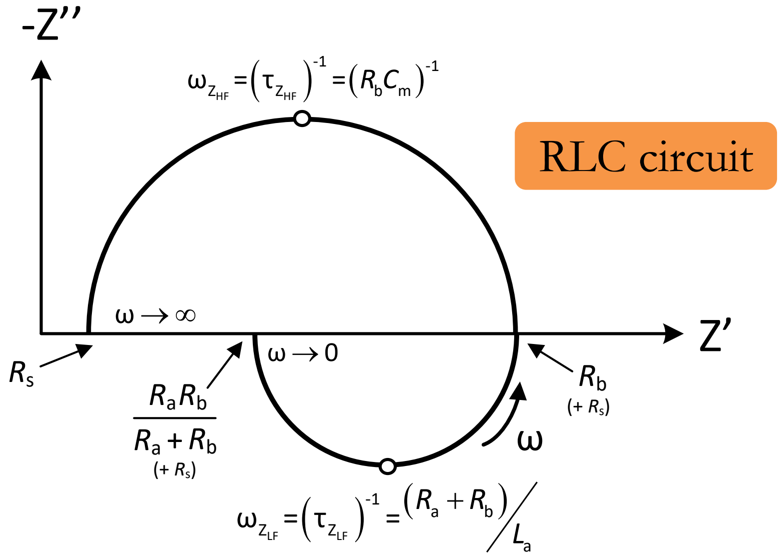


$$Z_L(\omega) = j\omega L$$







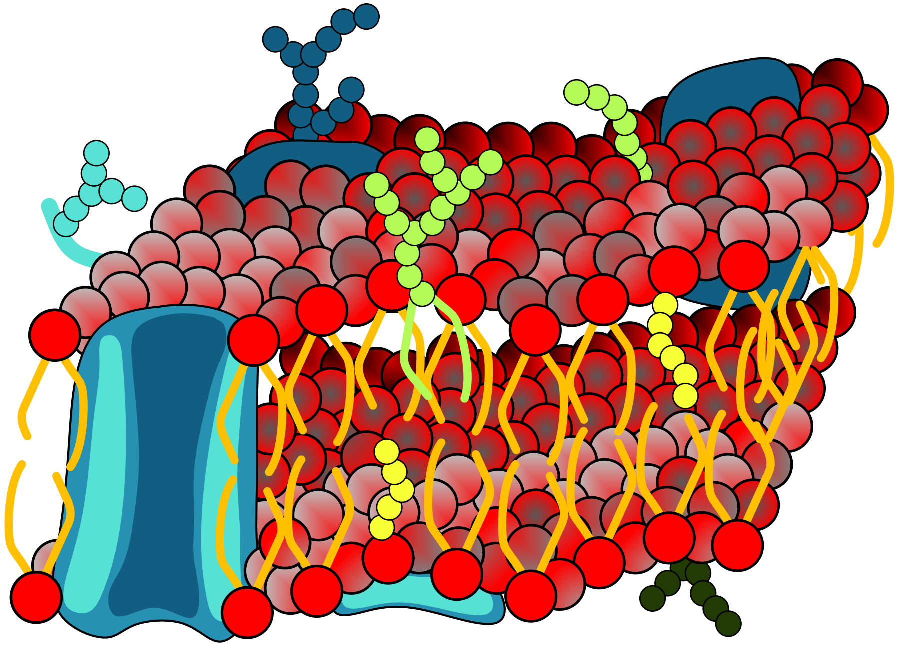


CELL MEMBRANE

Application in neurosciences

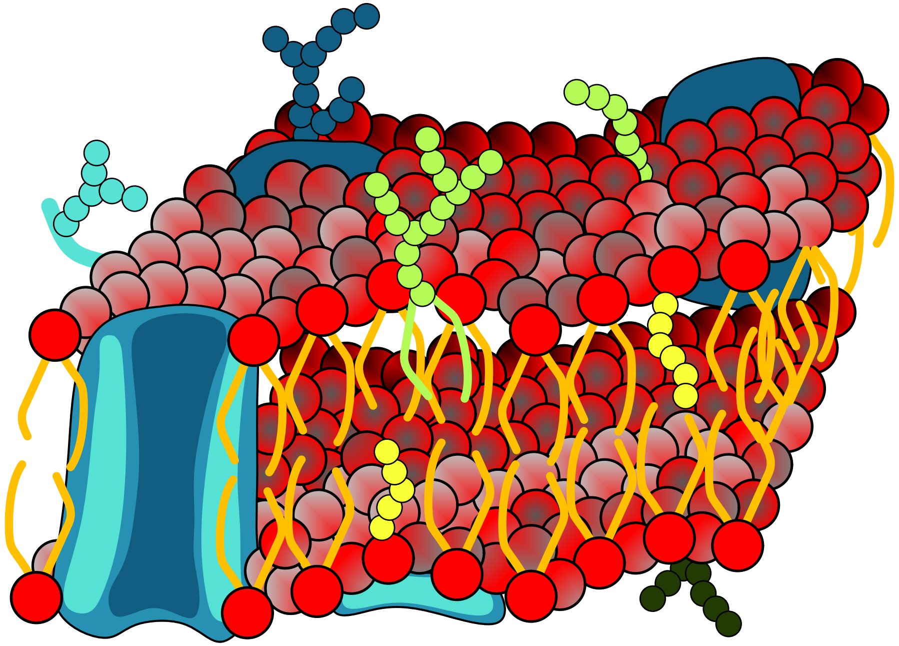
CELL MEMBRANE

Application in neurosciences



CELL MEMBRANE

Application in neurosciences

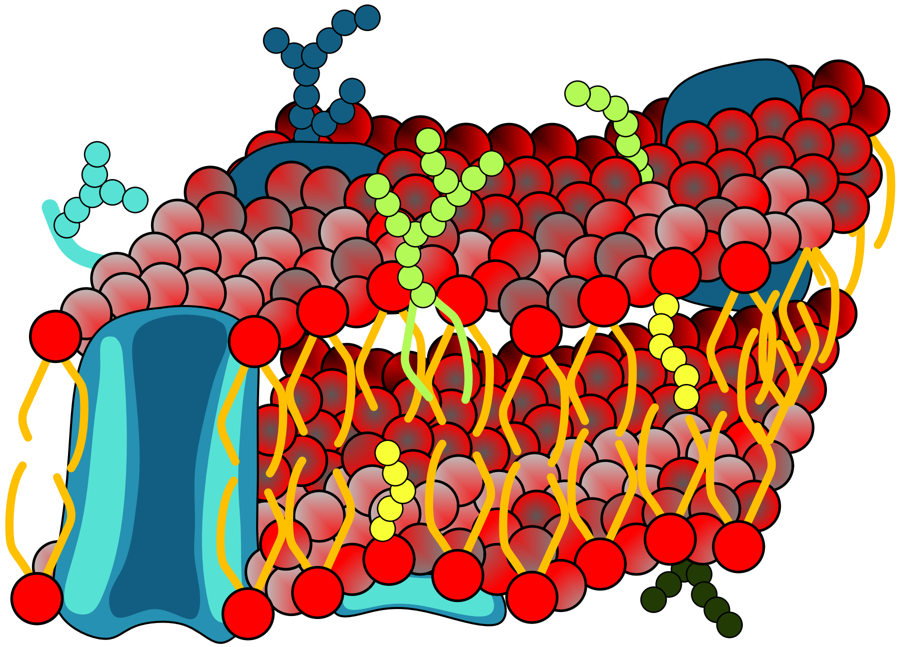


Membrane capacitance

Phospholipid bilayer acts as a dielectric wall that separates the charge that exists in the cytoplasm from that in the extracellular matrix

CELL MEMBRANE

Application in neurosciences



Membrane capacitance

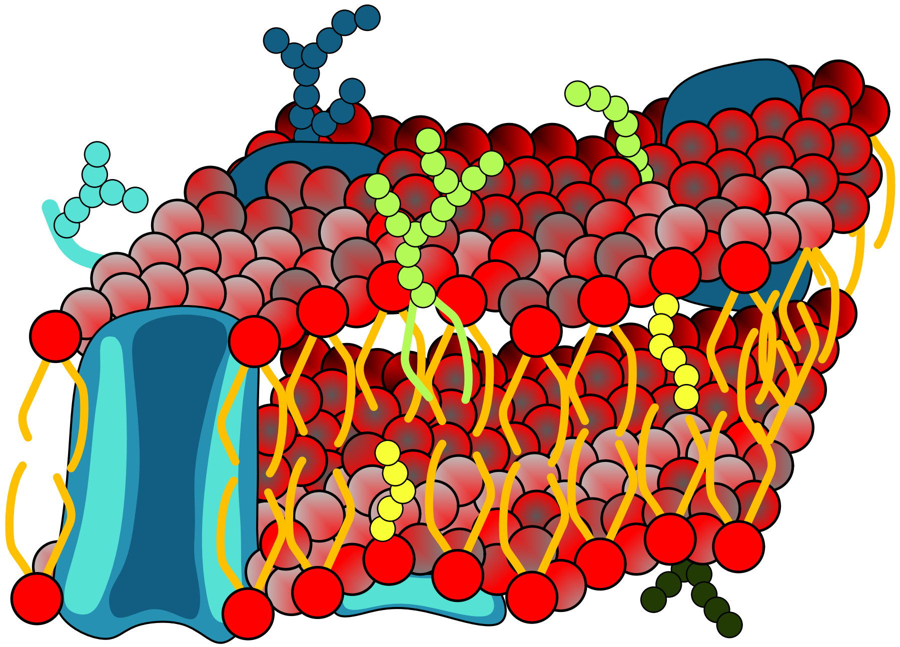
Phospholipid bilayer acts as a dielectric wall that separates the charge that exists in the cytoplasm from that in the extracellular matrix

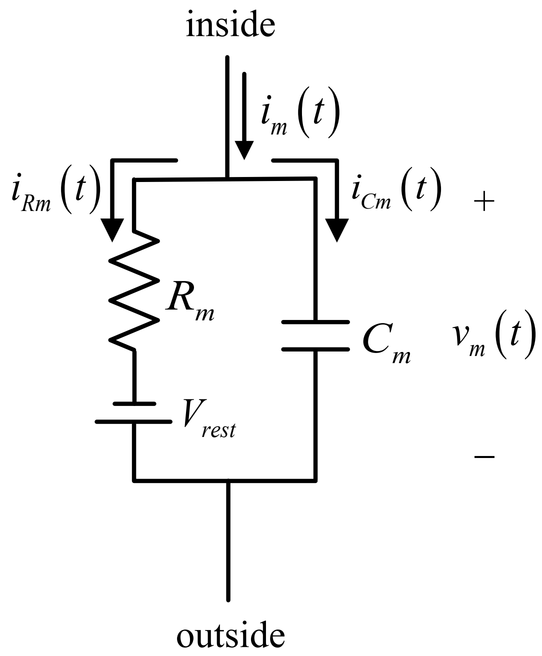
Membrane resistance

Selective permeabilities of the membrane to the different ionic species.

CELL MEMBRANE

Application in neurosciences





Membrane capacitance

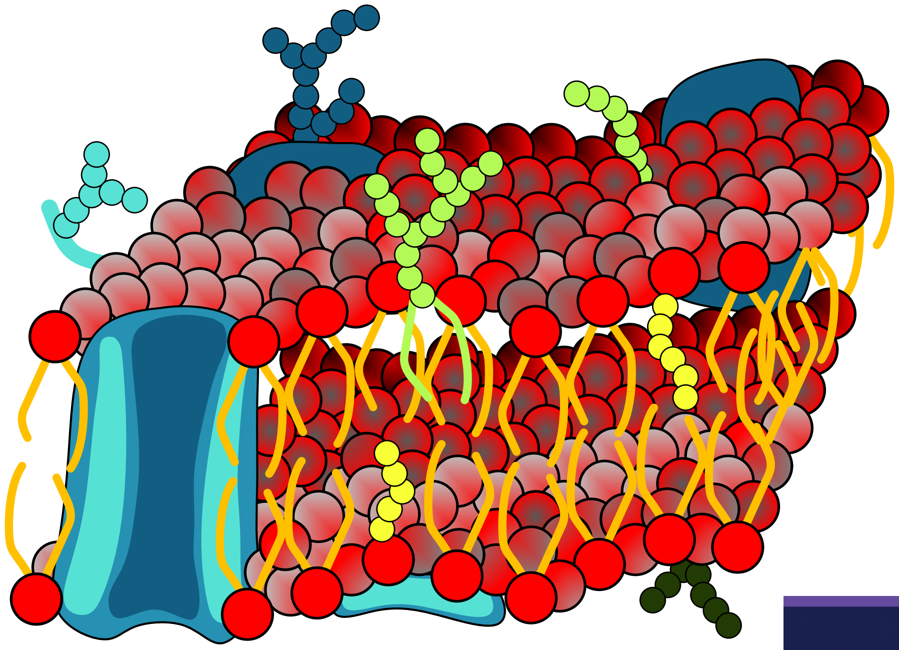
Phospholipid bilayer acts as a dielectric wall that separates the charge that exists in the cytoplasm from that in the extracellular matrix

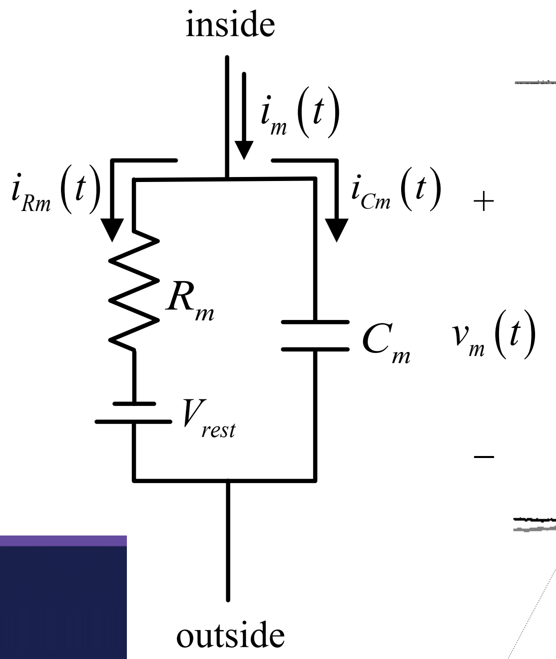
Membrane resistance

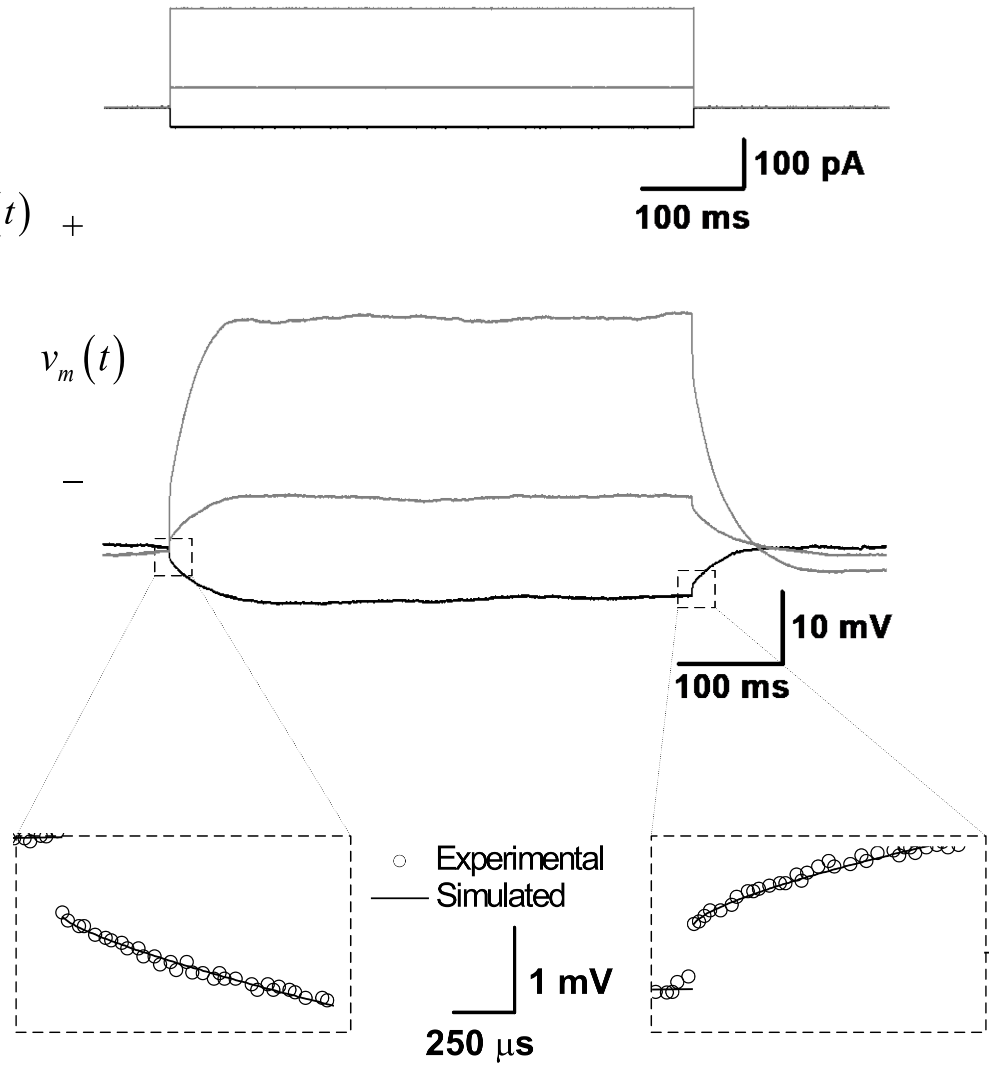
Selective permeabilities of the membrane to the different ionic species.

CELL MEMBRANE

Application in neurosciences







Membrane capacitance

Phospholipid bilayer acts as a dielectric wall that separates the charge that exists in the cytoplasm from that in the extracellular matrix

Membrane resistance

Selective permeabilities of the membrane to the different ionic species.

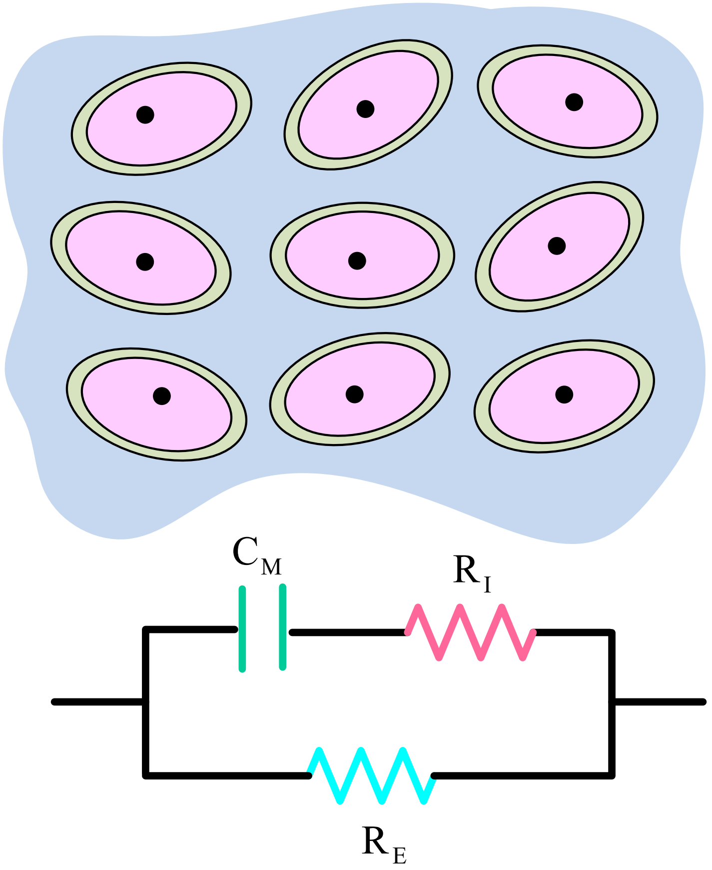
E. Hernández-Balaguera et al. *J. Electrochem. Society* 165(12) (2018) G3104

BIOLOGICAL TISSUE

Monitoring physiological states

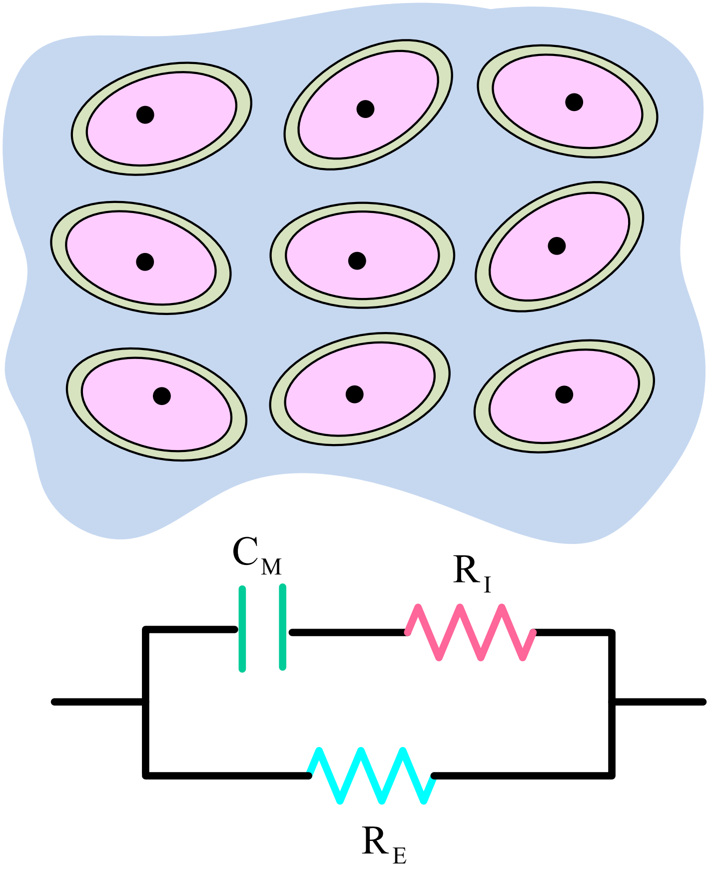
BIOLOGICAL TISSUE

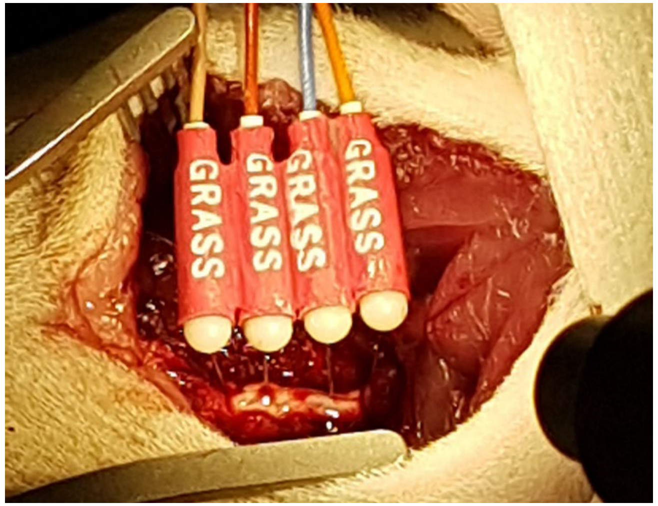
Monitoring physiological states



BIOLOGICAL TISSUE

Monitoring physiological states

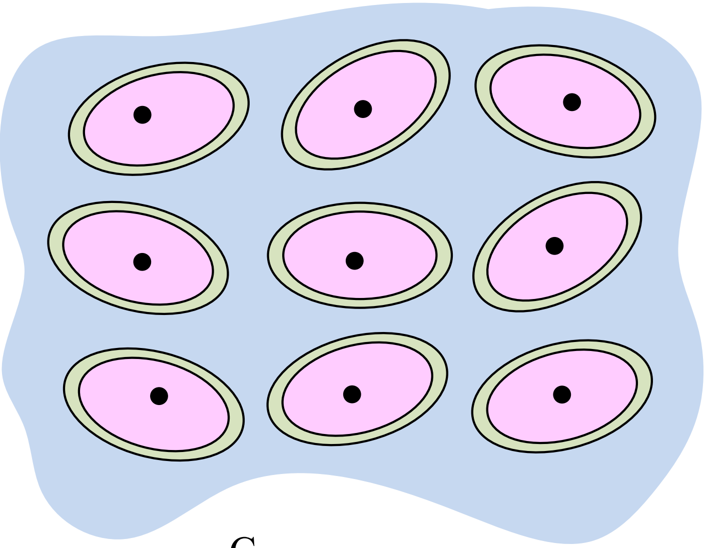


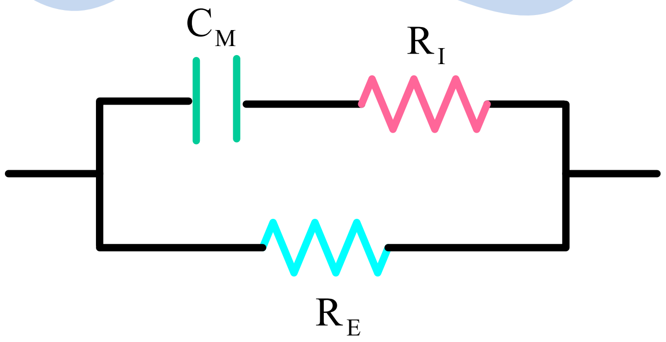


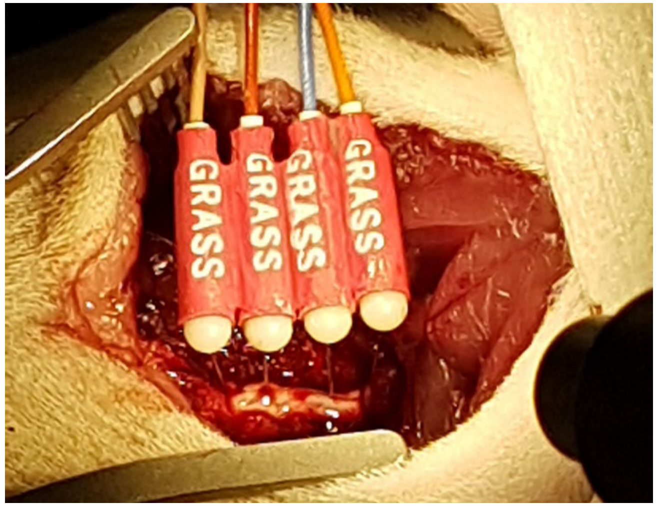
BIOLOGICAL TISSUE

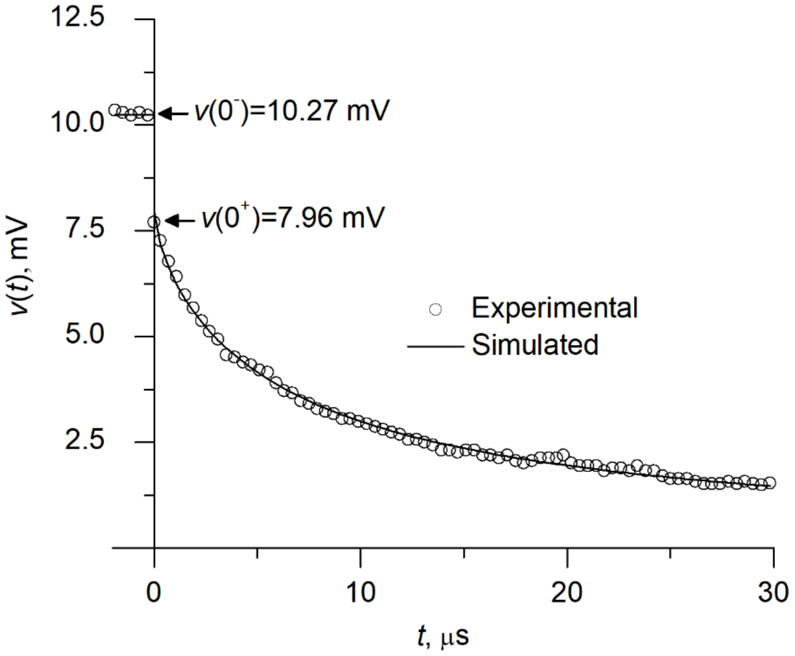
Monitoring physiological states

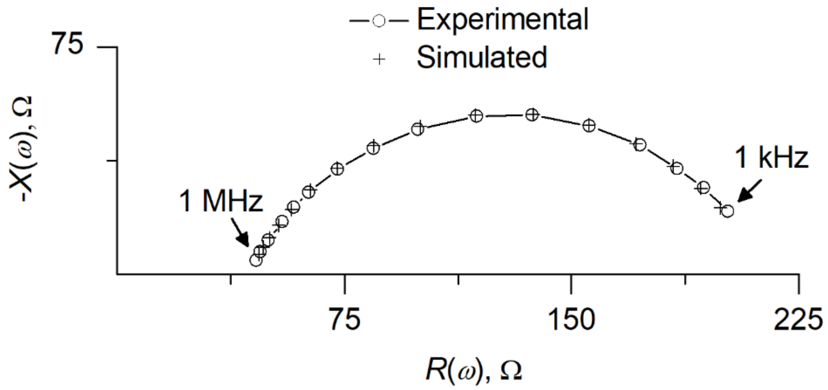
E. Hernández-Balaguera et al. *J. Electrochem. Society* 165(12) (2018) G3099







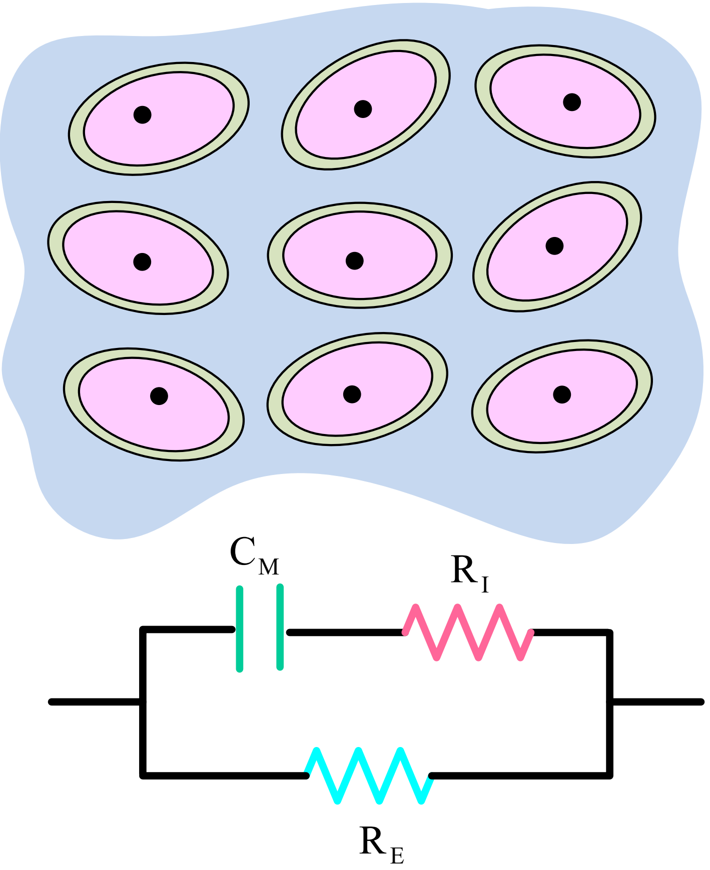


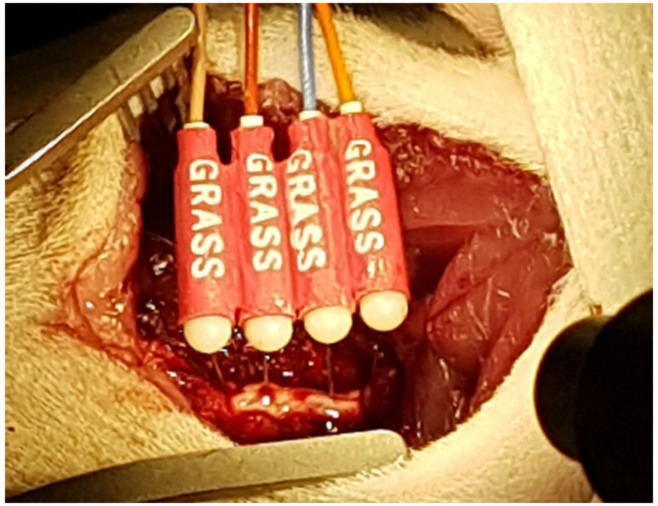


BIOLOGICAL TISSUE

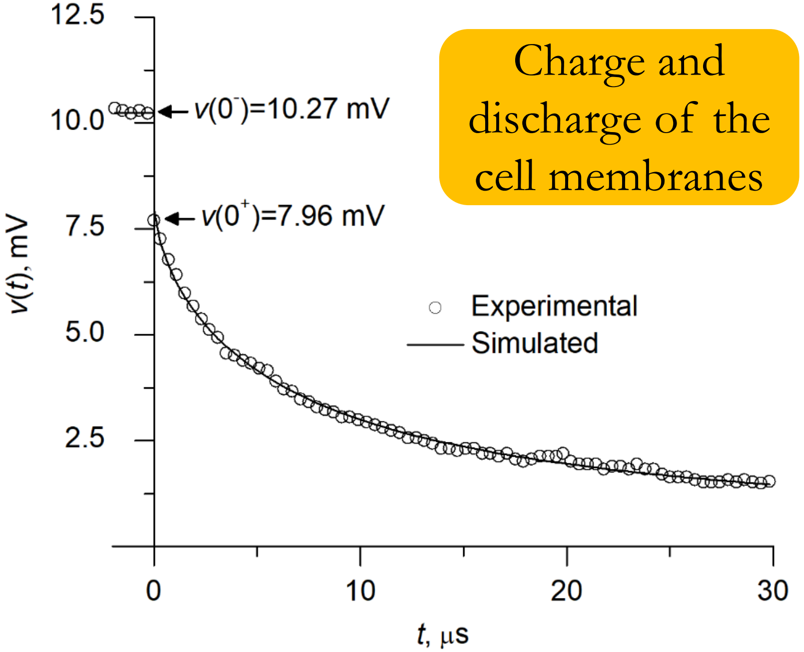
Monitoring physiological states

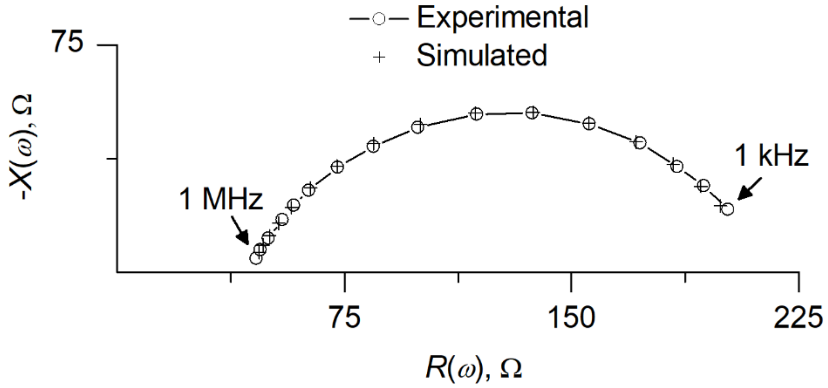
E. Hernández-Balaguera et al. *J. Electrochem. Society* 165(12) (2018) G3099





Charge and discharge of the cell membranes



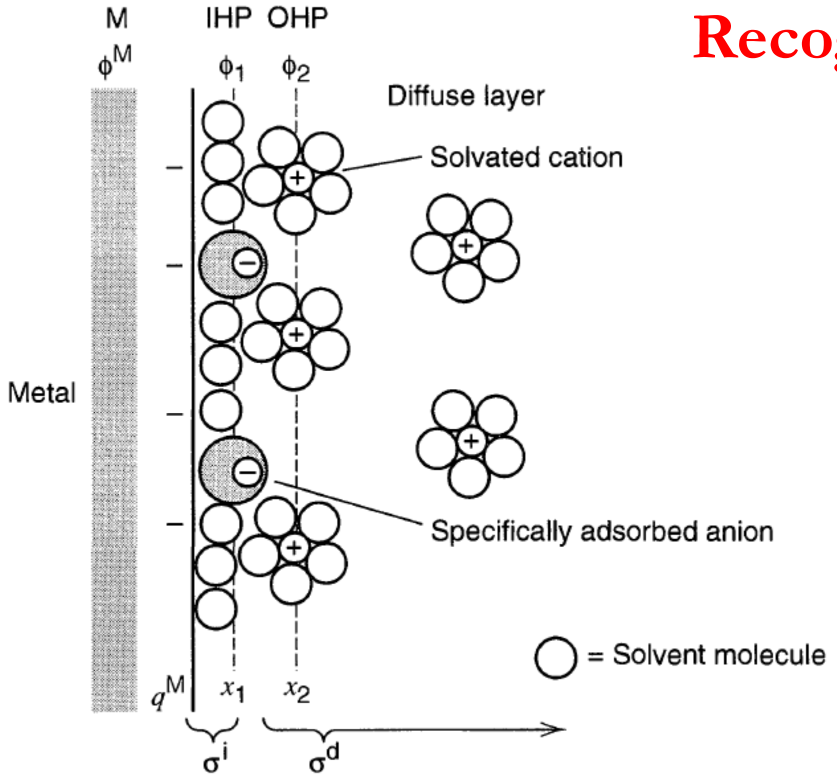


ELECTROCHEMISTRY

Recognition of interfacial events

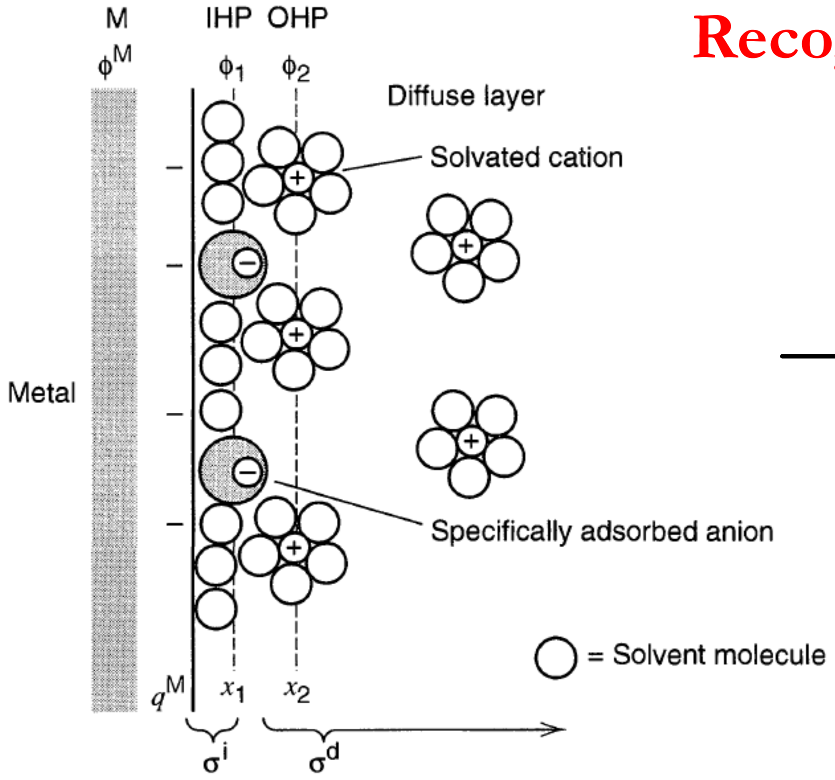
ELECTROCHEMISTRY

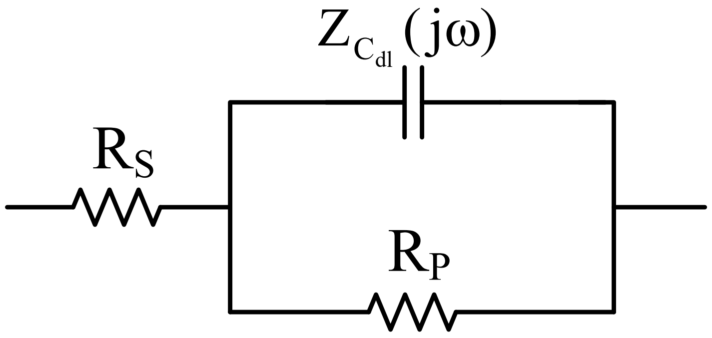
Recognition of interfacial events



ELECTROCHEMISTRY

Recognition of interfacial events

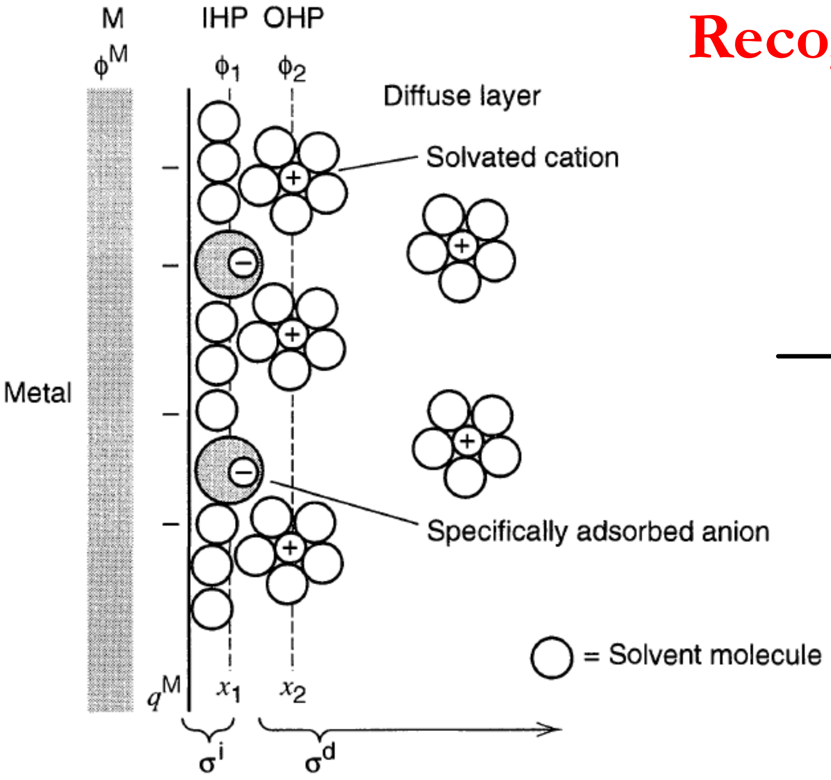


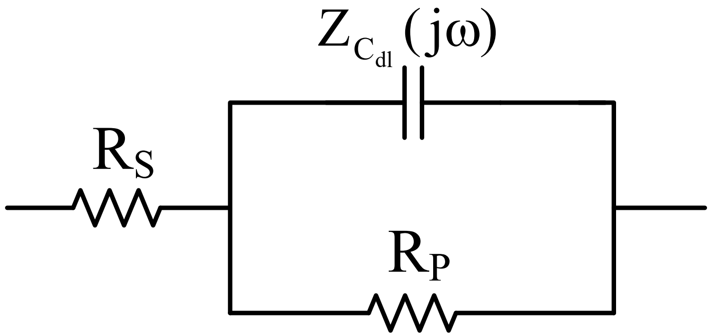


- C_{dl}: Double-layer capacitance
- R_p: Charge-transfer resistance

ELECTROCHEMISTRY

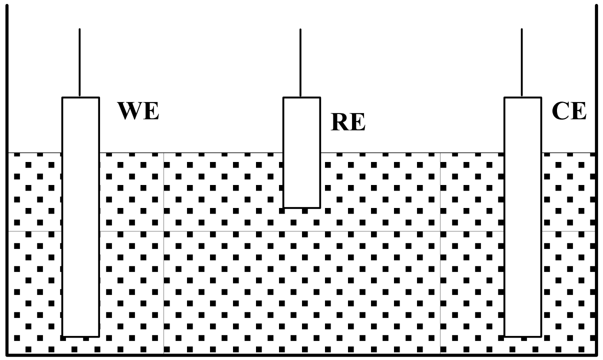
Recognition of interfacial events





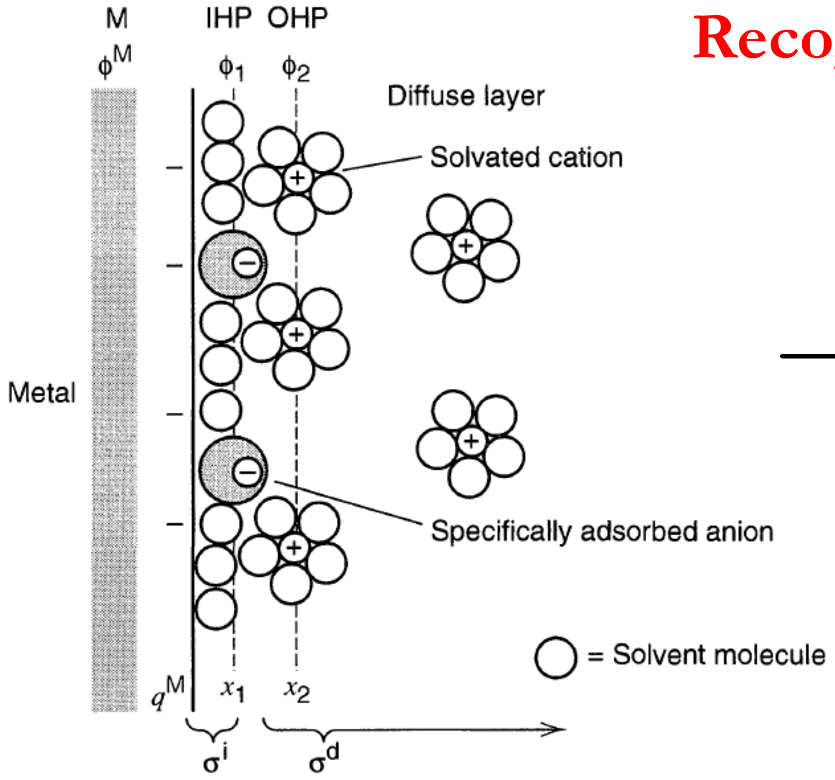
- C_{dl}: Double-layer capacitance
- R_p: Charge-transfer resistance

Three-electrode arrangement

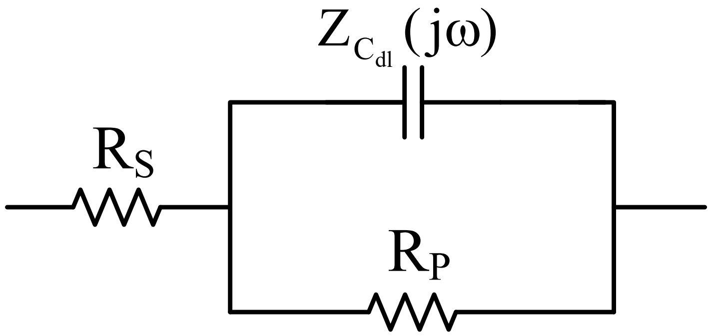


ELECTROCHEMISTRY

Recognition of interfacial events

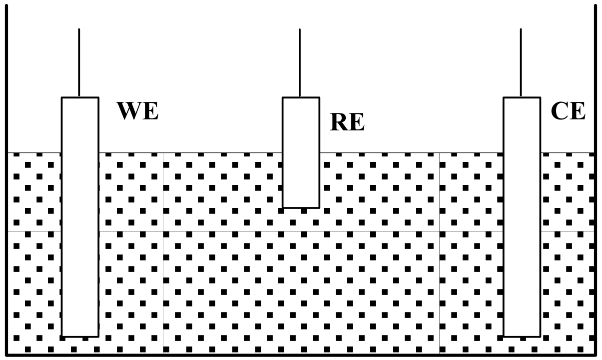


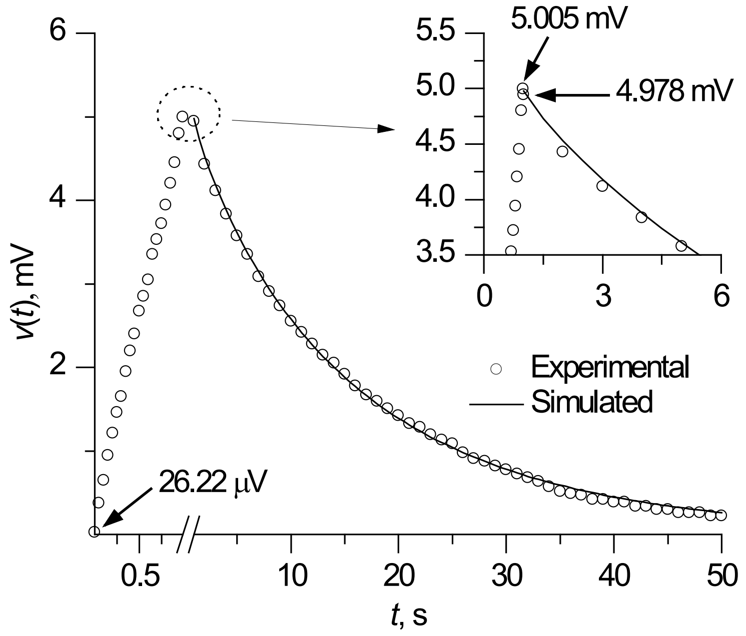
E. Hernández-Balaguera et al. *Electrochimica Acta* 233 (2017) 167

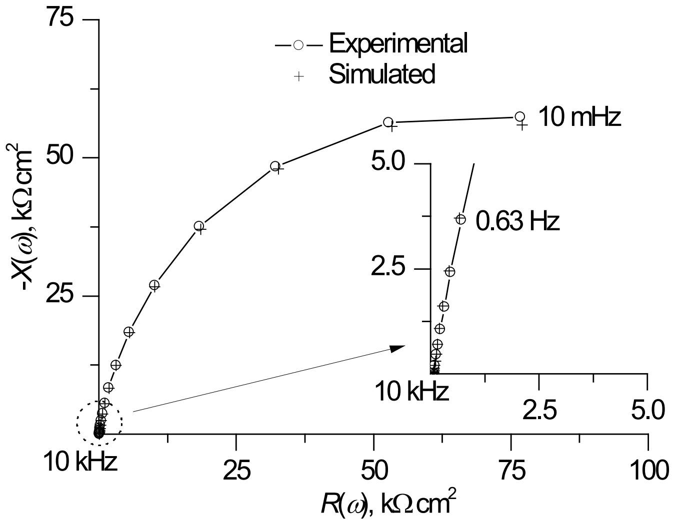


- C_{dl}: Double-layer capacitance
- R_p: Charge-transfer resistance

Three-electrode arrangement





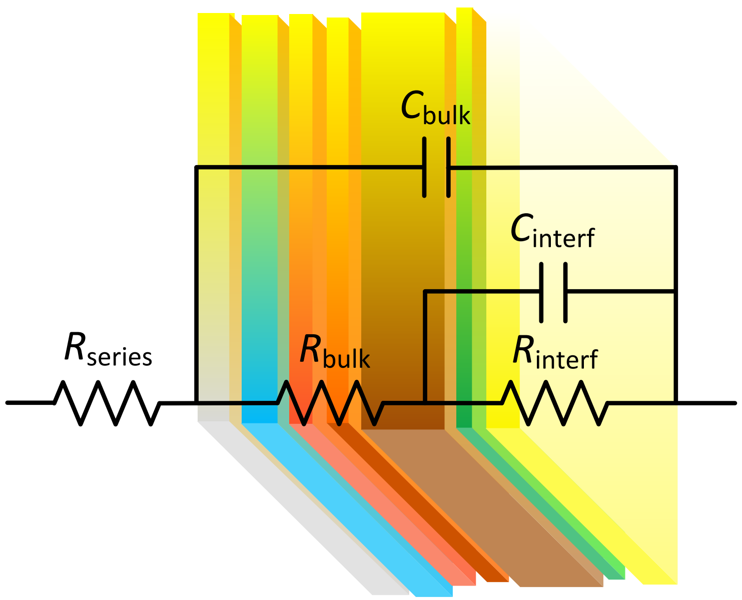


PHOTOVOLTAICS

Physical interpretation in solar cells

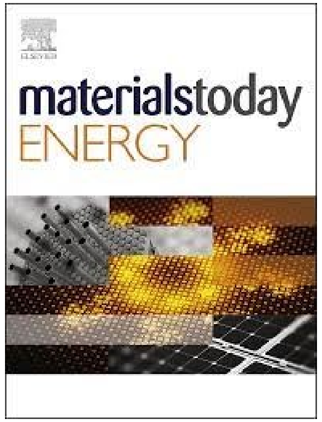
PHOTOVOLTAICS

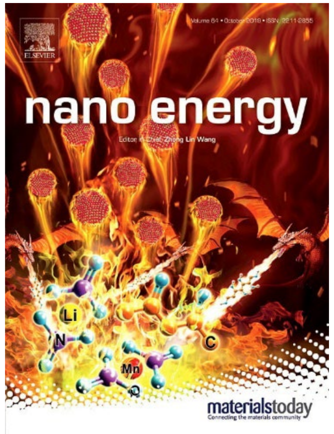
Physical interpretation in solar cells



PHOTOVOLTAICS

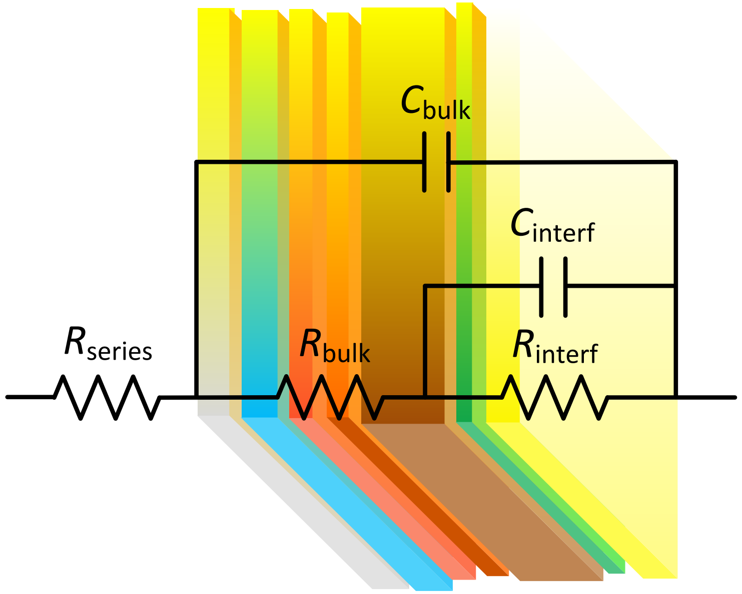
Physical interpretation in solar cells

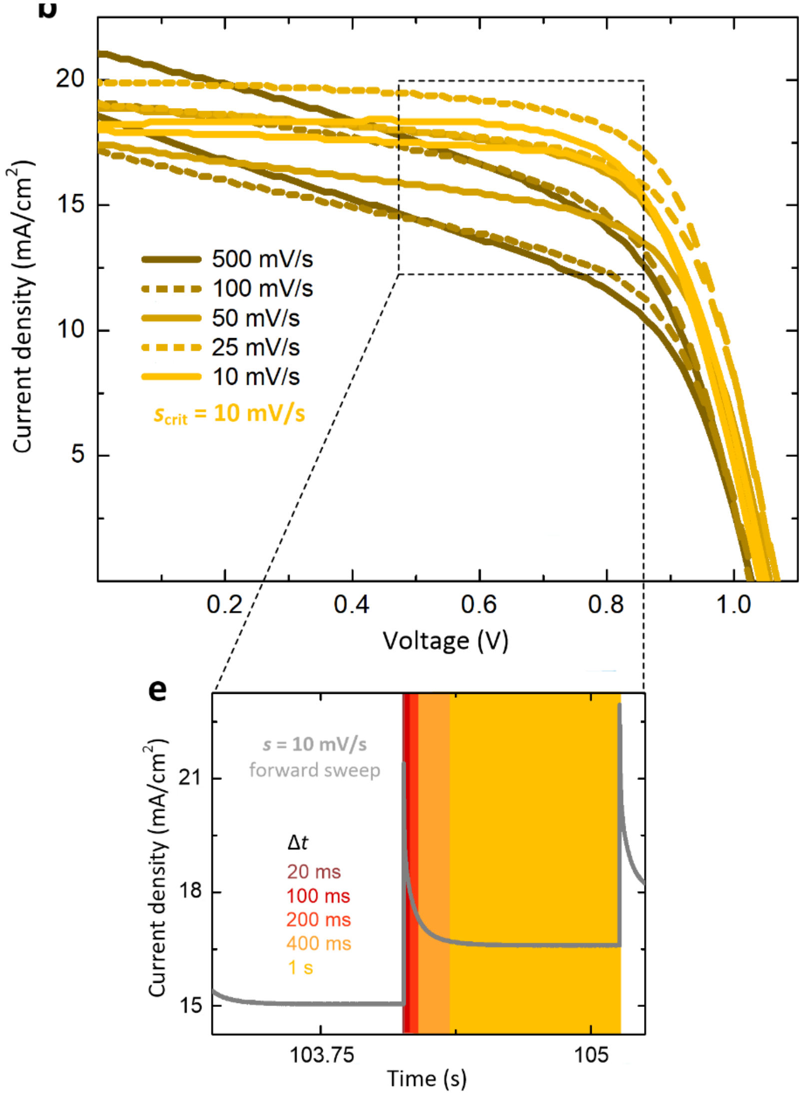




E. Hernández-Balaguera et al. *Materials Today Energy* 27 (2022) 101031

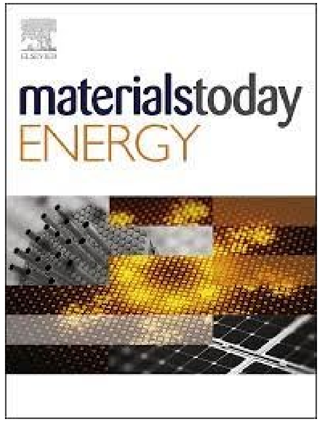
E. Hernández-Balaguera et al. *Nano Energy* 78 (2020) 105398

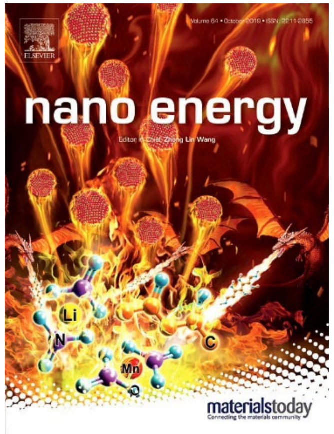




PHOTOVOLTAICS

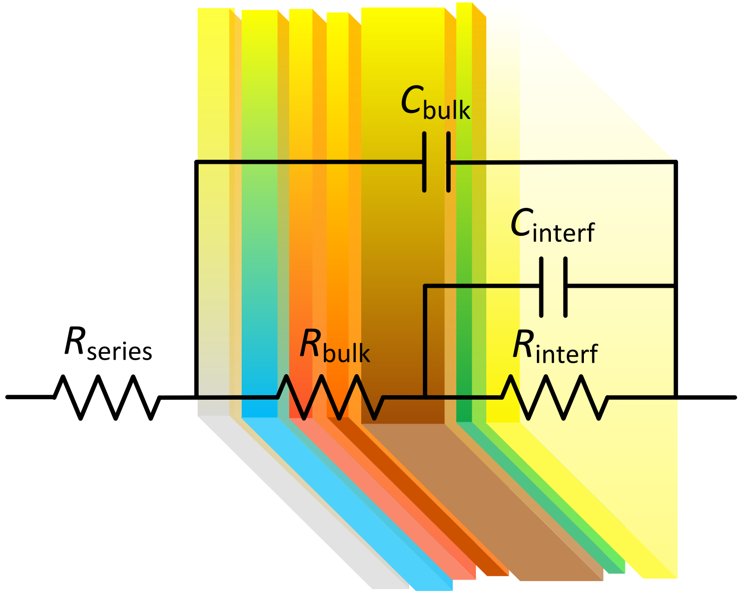
Physical interpretation in solar cells

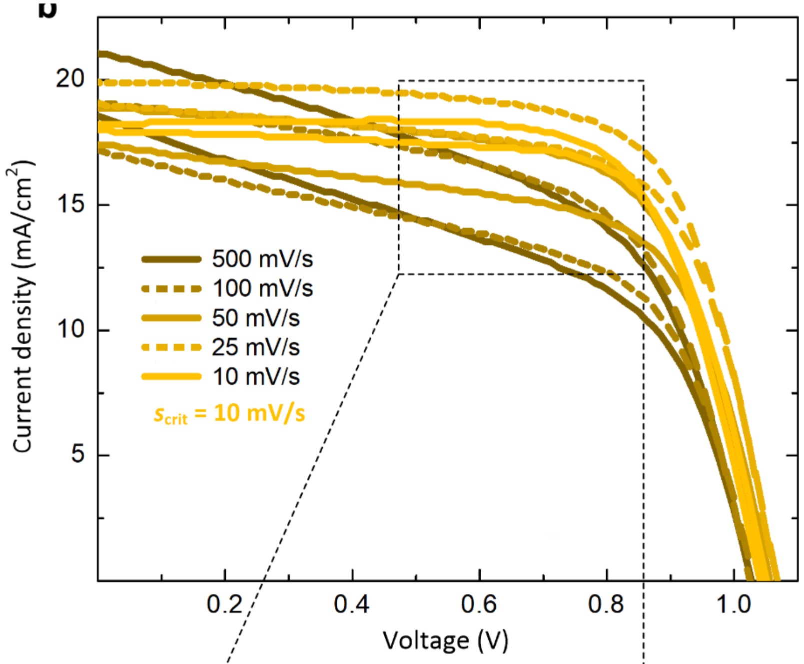


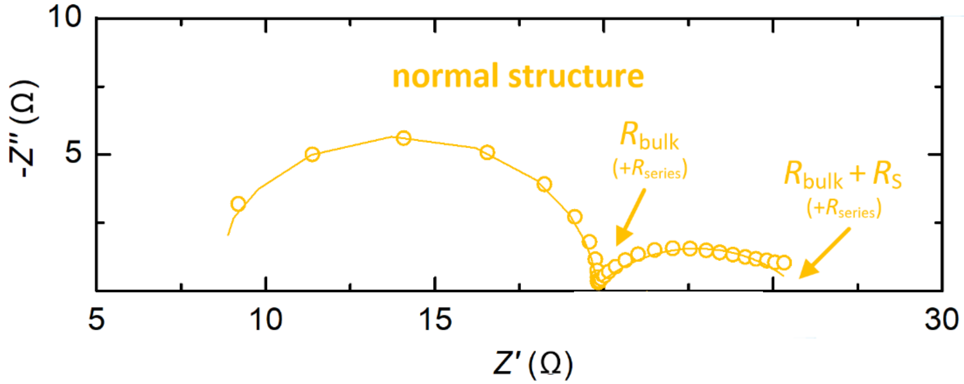


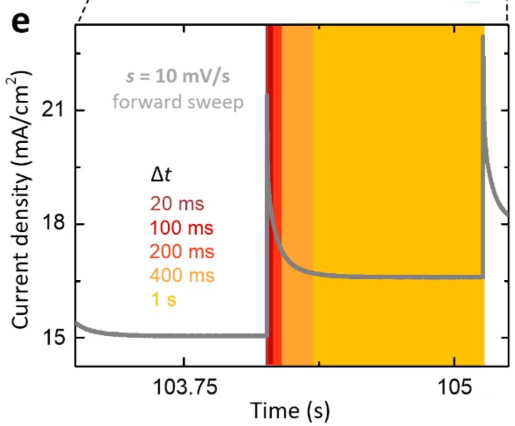
E. Hernández-Balaguera et al. *Materials Today Energy* 27 (2022) 101031

E. Hernández-Balaguera et al. *Nano Energy* 78 (2020) 105398



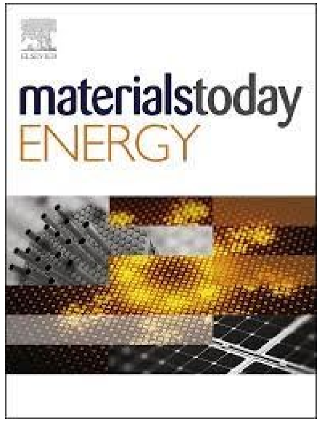


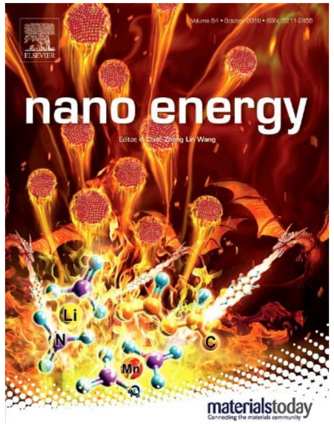




PHOTOVOLTAICS

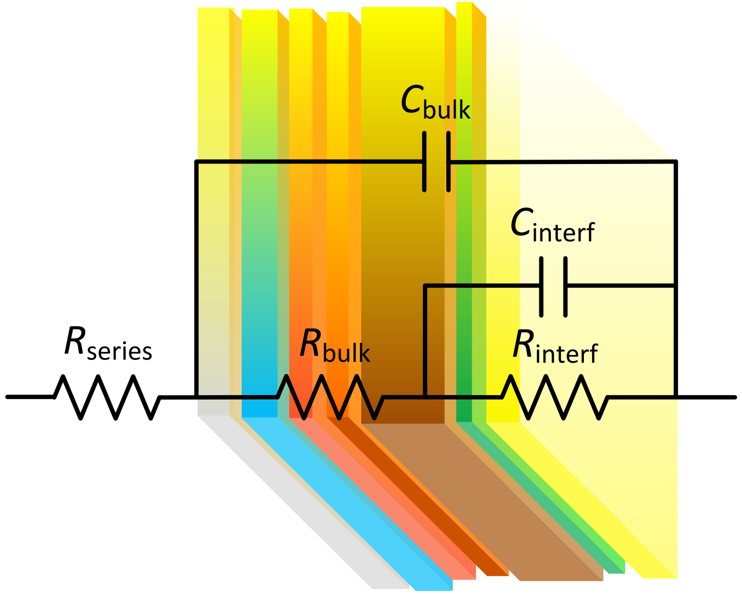
Physical interpretation in solar cells

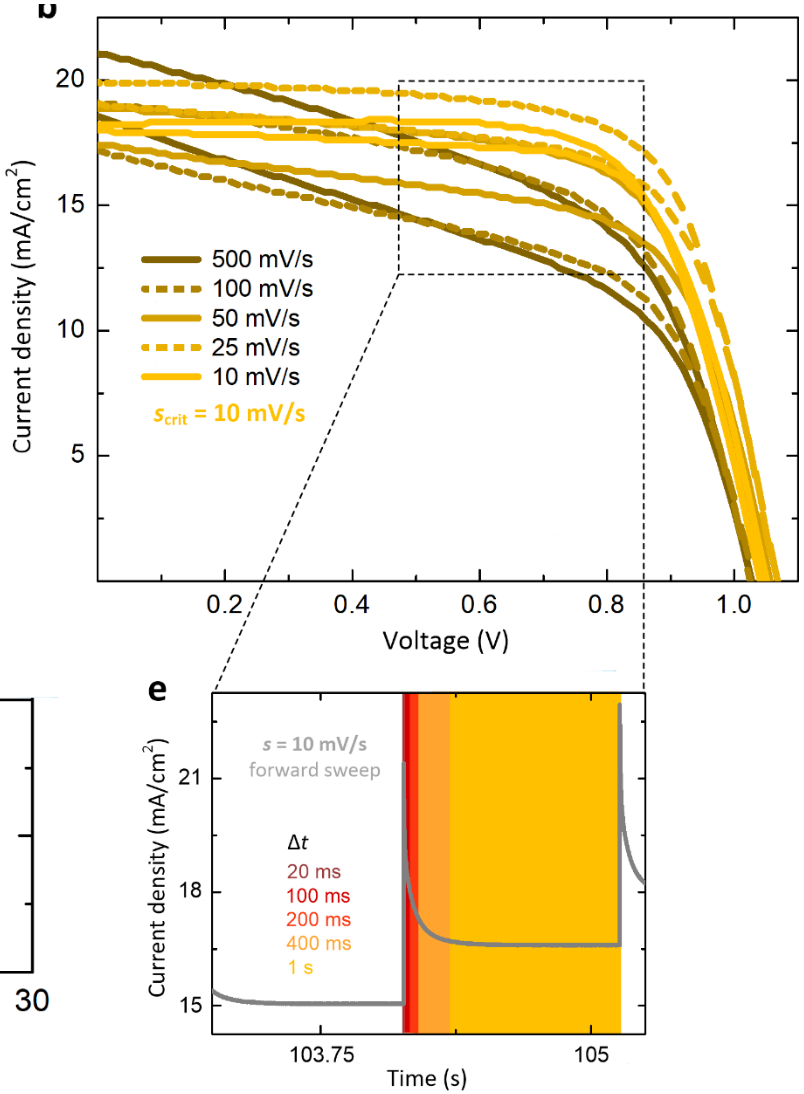




E. Hernández-Balaguera et al. *Materials Today Energy* 27 (2022) 101031

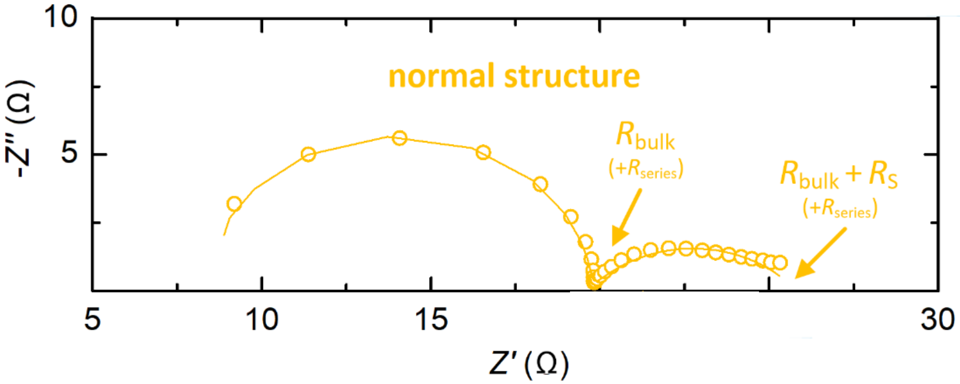
E. Hernández-Balaguera et al. *Nano Energy* 78 (2020) 105398





Recombination and accumulation of charge processses

Voltage-dependent mechanisms

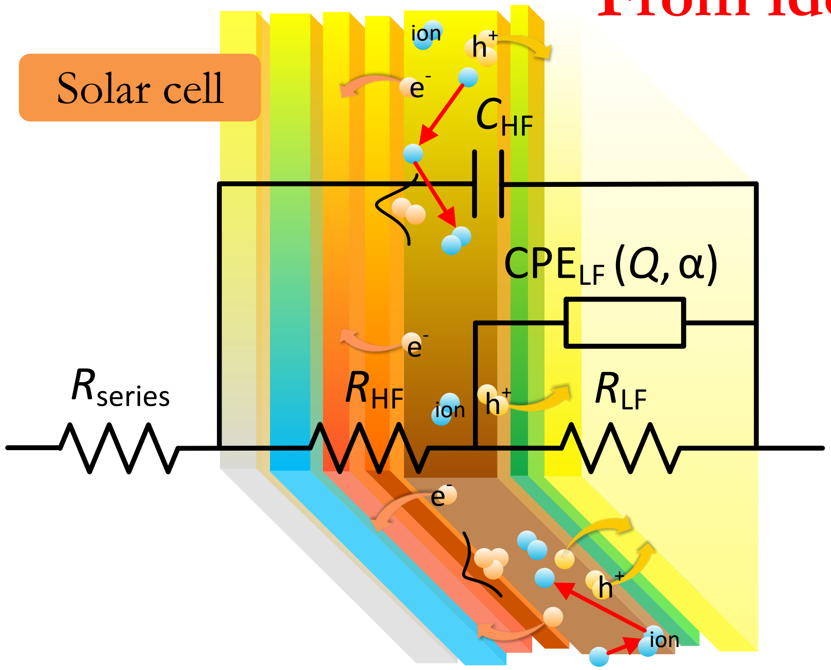


IN THE “REAL WORLD”

From ideal to anomalous dynamics

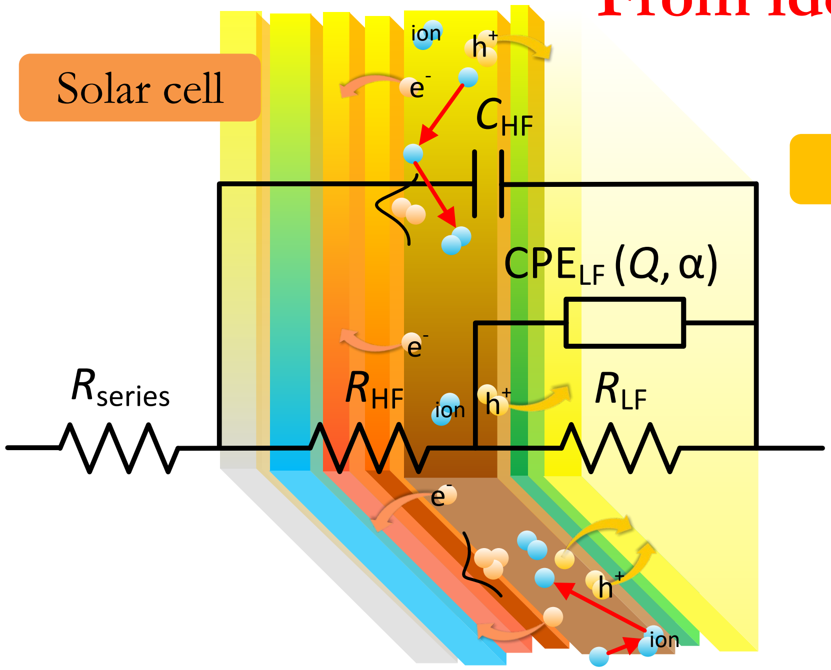
IN THE "REAL WORLD"

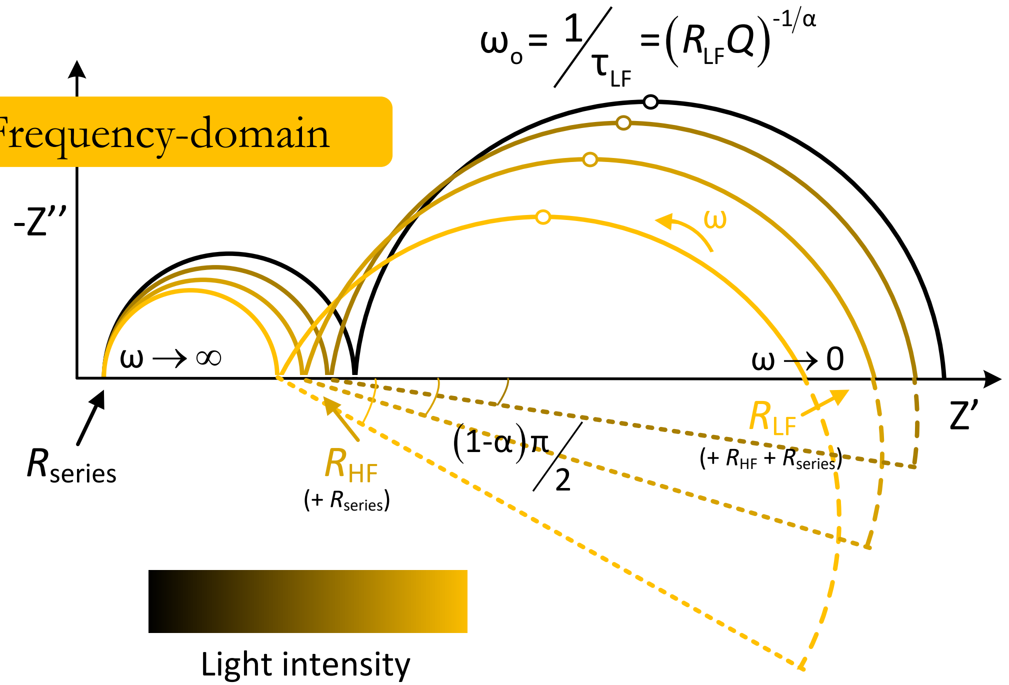
From ideal to anomalous dynamics



IN THE "REAL WORLD"

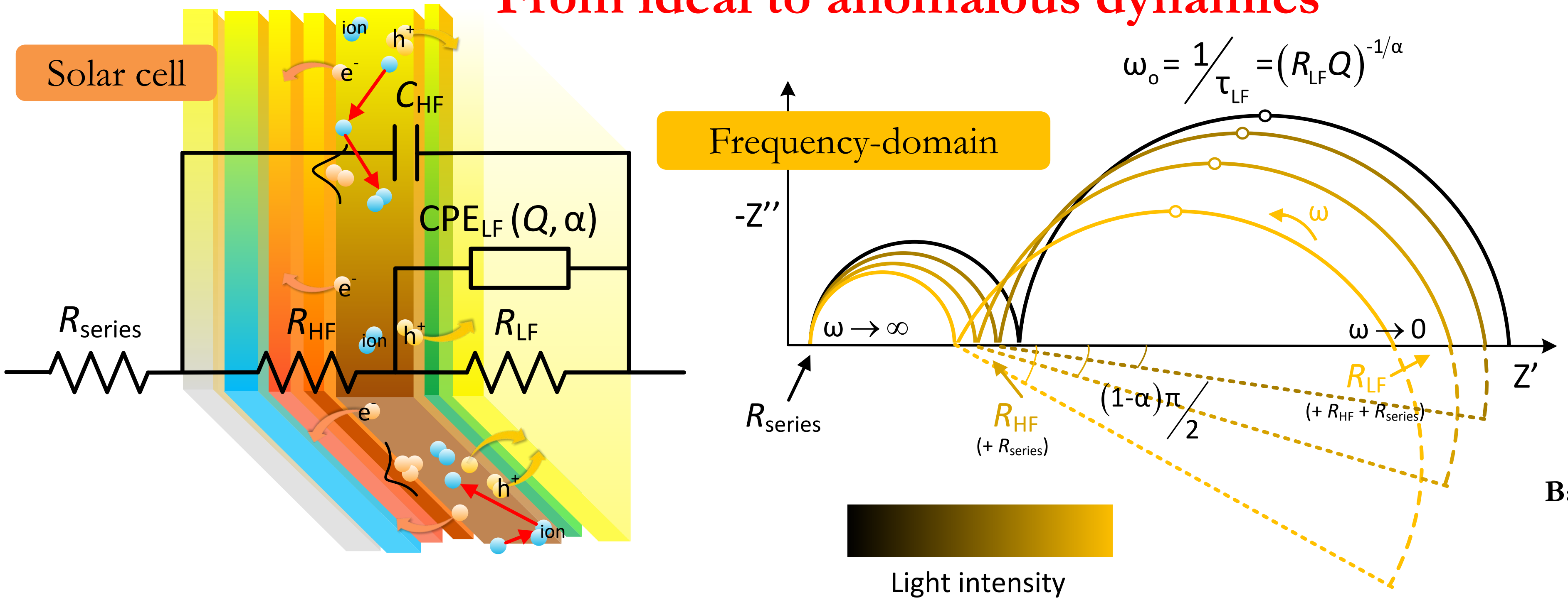
From ideal to anomalous dynamics

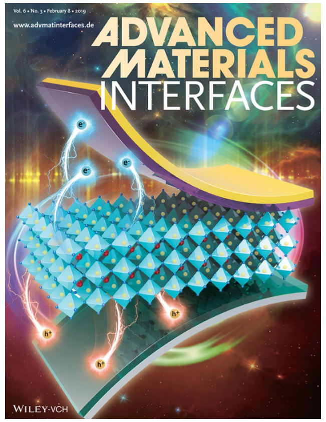




IN THE "REAL WORLD"

From ideal to anomalous dynamics

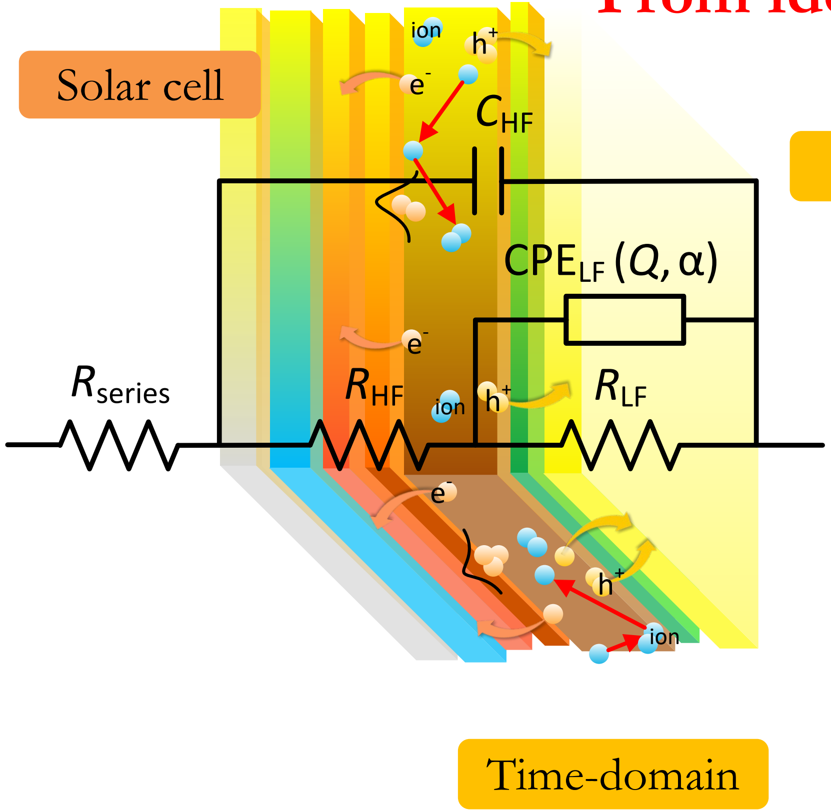


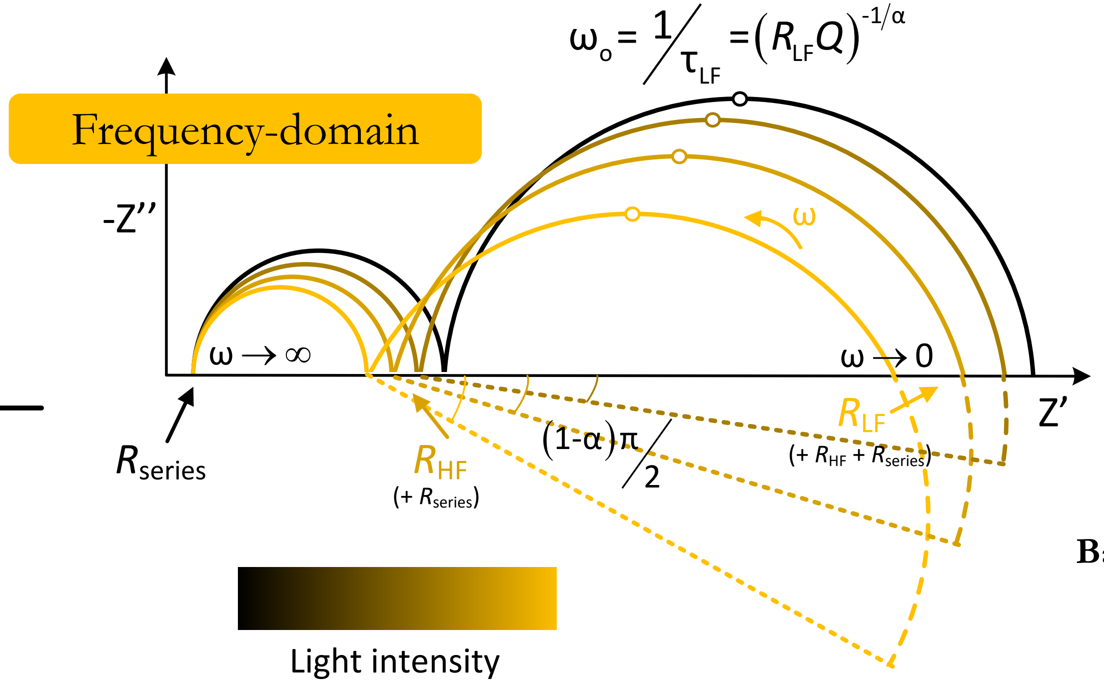


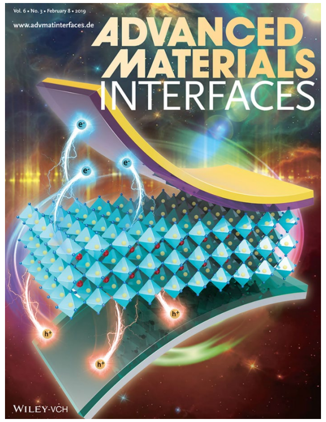
E. Hernández-Balaguera et al. *Advanced Materials Interfaces* 9(9) (2022) 2102275

IN THE “REAL WORLD”

From ideal to anomalous dynamics



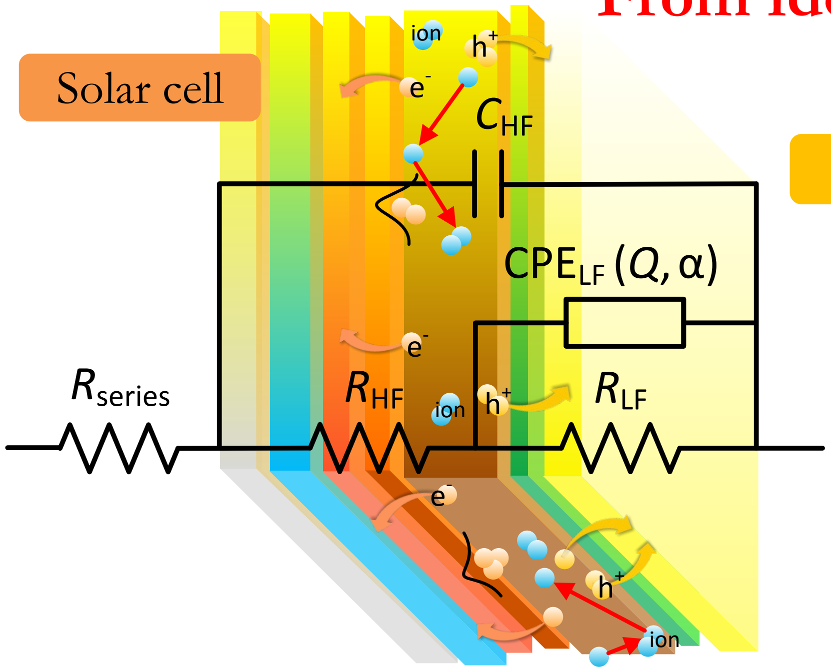


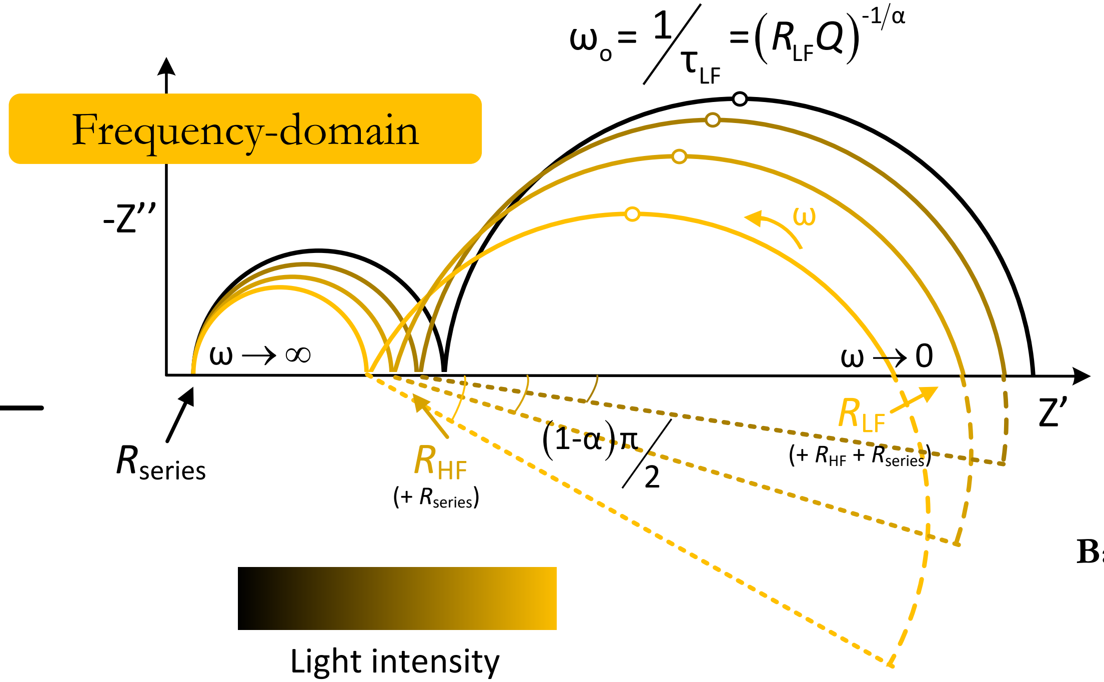


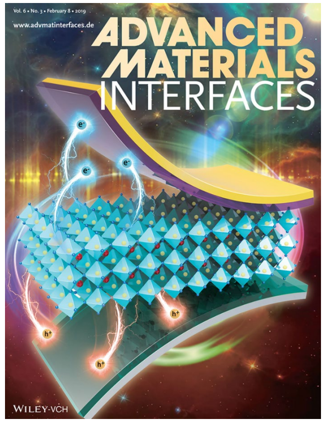
E. Hernández-Balaguera et al. *Advanced Materials Interfaces* 9(9) (2022) 2102275

IN THE “REAL WORLD”

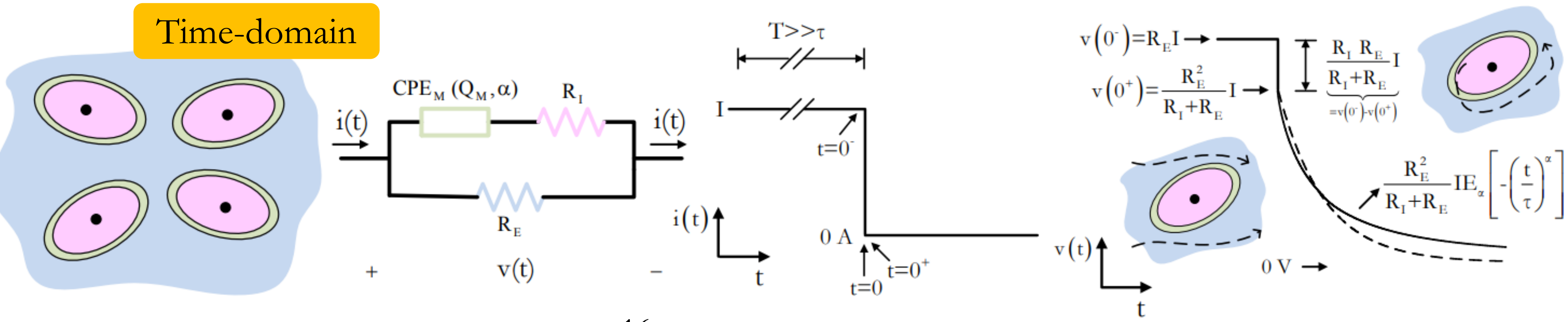
From ideal to anomalous dynamics





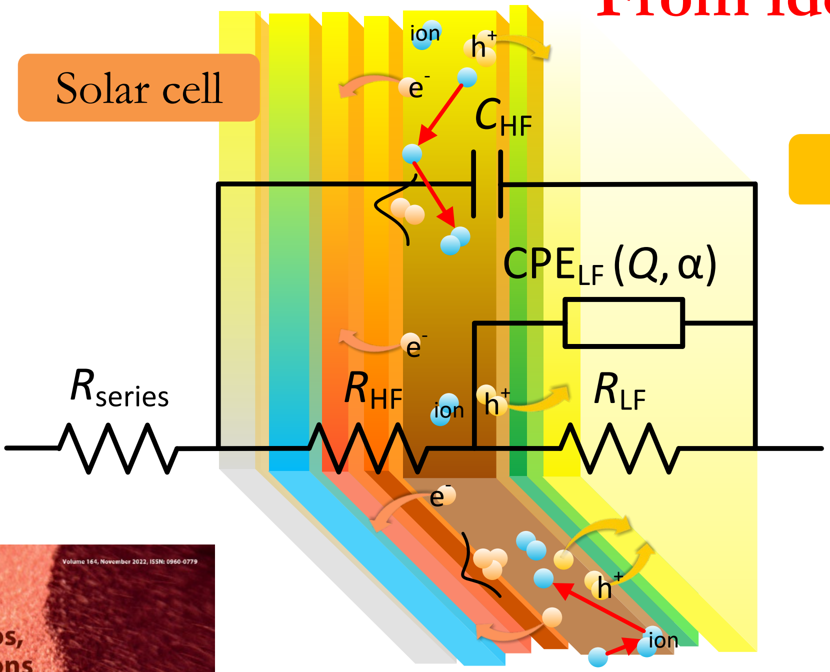


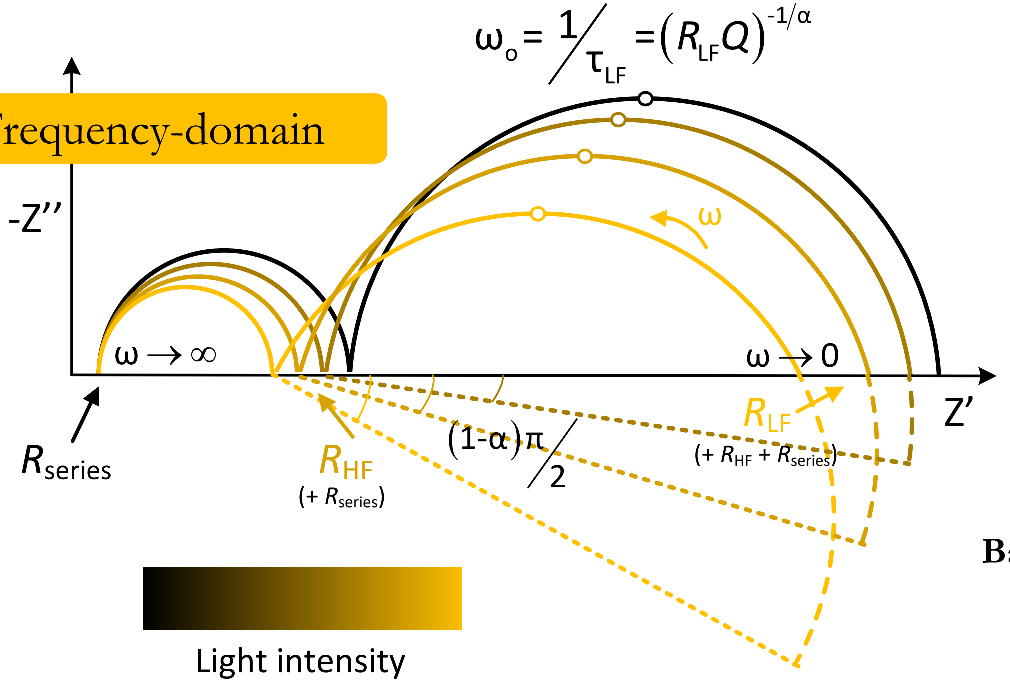
E. Hernández-Balaguera et al. *Advanced Materials Interfaces* 9(9) (2022) 2102275

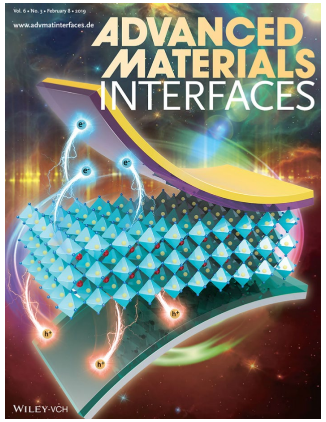


IN THE "REAL WORLD"

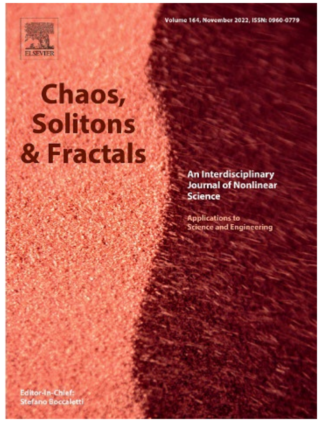
From ideal to anomalous dynamics



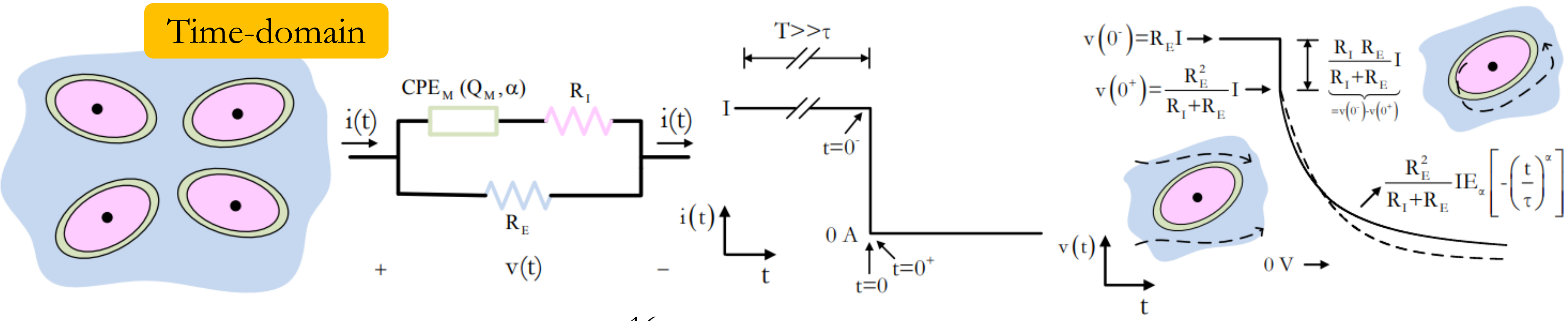




E. Hernández-Balaguera et al. *Advanced Materials Interfaces* 9(9) (2022) 2102275



E. Hernández-Balaguera et al. *Chaos, Solitons & Fractals* 145 (2022) 110768



CONSTANT PHASE ELEMENT (CPE)

Introduction

CONSTANT PHASE ELEMENT (CPE)

Introduction

- **Distributed-parameter network:** Modeling of real-world physical systems that occupy a finite region of space and their electrical properties are "distributed" → Time-constants distribution.

CONSTANT PHASE ELEMENT (CPE)

Introduction

- **Distributed-parameter network:** Modeling of real-world physical systems that occupy a finite region of space and their electrical properties are “distributed” → Time-constants distribution.
- Circuit element very used in “Network analysis and synthesis”.

CONSTANT PHASE ELEMENT (CPE)

Introduction

- **Distributed-parameter network:** Modeling of real-world physical systems that occupy a finite region of space and their electrical properties are "distributed" → Time-constants distribution.
- Circuit element very used in "Network analysis and synthesis".
- **Characteristic equations:**
    - Frequency-domain:

$$Z(j\omega)=\frac{1}{Q(j\omega)^{\alpha}}$$

CONSTANT PHASE ELEMENT (CPE)

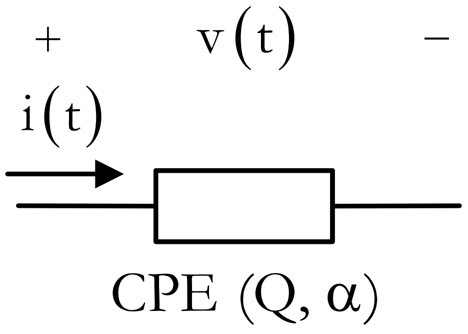
Introduction

- **Distributed-parameter network:** Modeling of real-world physical systems that occupy a finite region of space and their electrical properties are "distributed" → Time-constants distribution.
- Circuit element very used in "Network analysis and synthesis".
- **Characteristic equations:**
  - Frequency-domain:

$$Z(j\omega)=\frac{1}{Q(j\omega)^{\alpha}}$$

  - Time-domain:

$$i(t)=Q\frac{d^{\alpha}v(t)}{dt^{\alpha}}$$



CONSTANT PHASE ELEMENT (CPE)

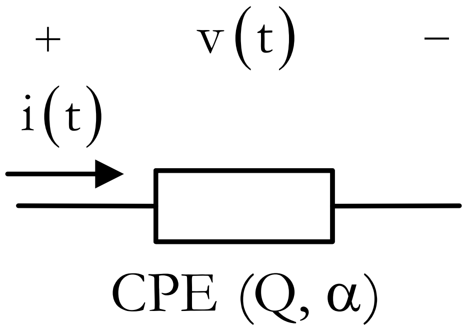
Introduction

- **Distributed-parameter network:** Modeling of real-world physical systems that occupy a finite region of space and their electrical properties are "distributed" → Time-constants distribution.
- Circuit element very used in "Network analysis and synthesis".
- **Characteristic equations:**
  - Frequency-domain:

$$Z(j\omega)=\frac{1}{Q(j\omega)^{\alpha}}$$

  - Time-domain:

$$i(t)=Q\frac{d^{\alpha}v(t)}{dt^{\alpha}}$$



➢ **Parameters and units:**

- Q: CPE parameter [Ω^{-1} s^{α}].
- α: CPE exponent [Dimensionless], $-1\leq\alpha\leq1$.

- **α = 0:** Resistance, R=1/Q.
- **α = 1:** Pure capacitance, C=Q.
- **α = -1:** Inductance, L=1/Q.
- Intermediate values:
  - **α = 0.5:** Warburg impedance (infinite RC transmission line with uniform distribution).

CONSTANT PHASE ELEMENT (CPE)

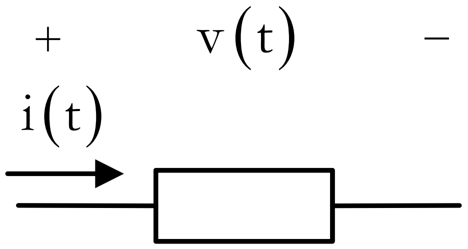
Introduction

- **Distributed-parameter network:** Modeling of real-world physical systems that occupy a finite region of space and their electrical properties are "distributed" → Time-constants distribution.
- Circuit element very used in "Network analysis and synthesis".
- **Characteristic equations:**
  - Frequency-domain:

$$Z(j\omega)=\frac{1}{Q(j\omega)^{\alpha}}$$

  - Time-domain:

$$i(t)=Q\frac{d^{\alpha}v(t)}{dt^{\alpha}}$$



CPE (Q, α)

- **Parameters and units:**
  - Q: CPE parameter [Ω^{-1} s^{α}].
  - α: CPE exponent [Dimensionless], $-1\leq\alpha\leq1$.

- **α = 0:** Resistance, R=1/Q.
- **α = 1:** Pure capacitance, C=Q.
- **α = -1:** Inductance, L=1/Q.
- Intermediate values:
  - **α = 0.5:** Warburg impedance (infinite RC transmission line with uniform distribution).

In the analysis of the electrical behavior of natural systems, it is generally considered a fractional (or non-integer order) capacitor (0<α<1).

Q: CPE parameter [F $s^{\alpha-1}$]

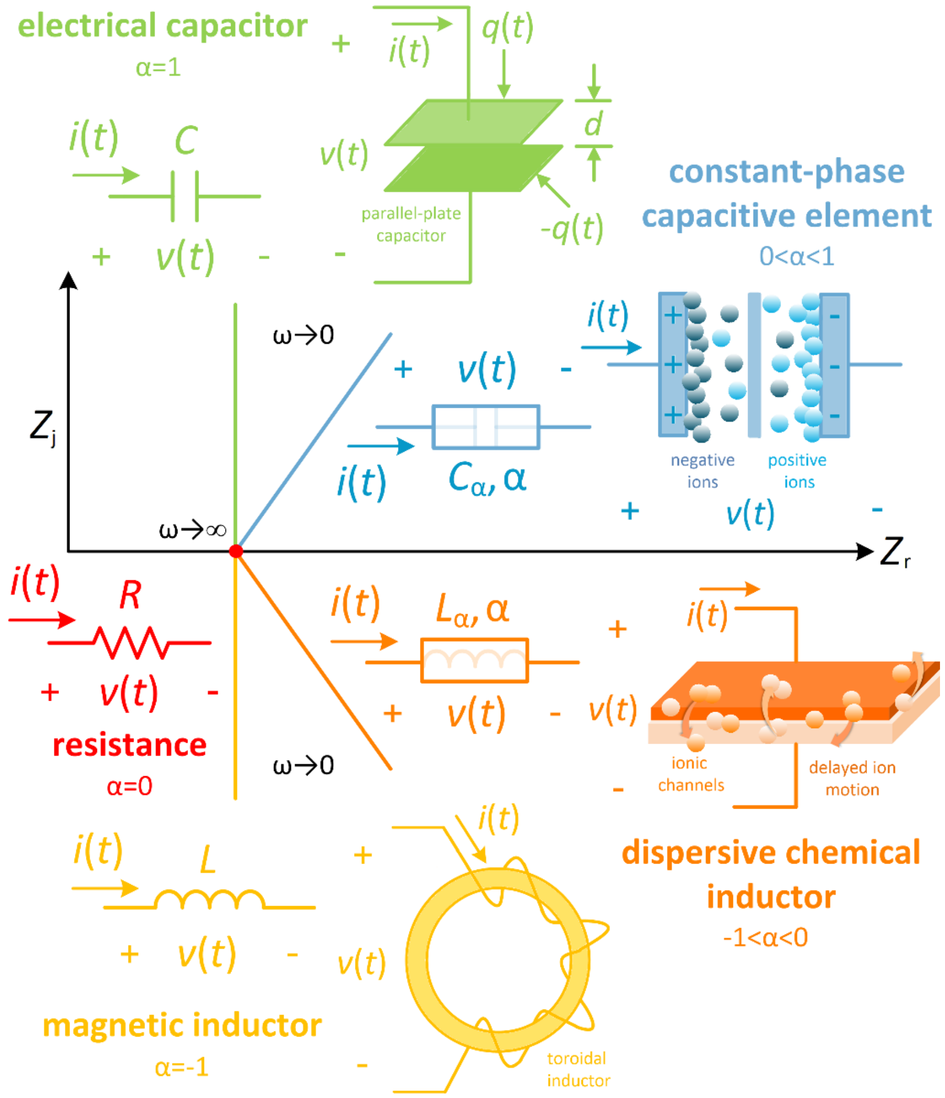
CONSTANT PHASE ELEMENT (CPE)

Origin: Frequency-response

CONSTANT PHASE ELEMENT (CPE)

Origin: Frequency-response

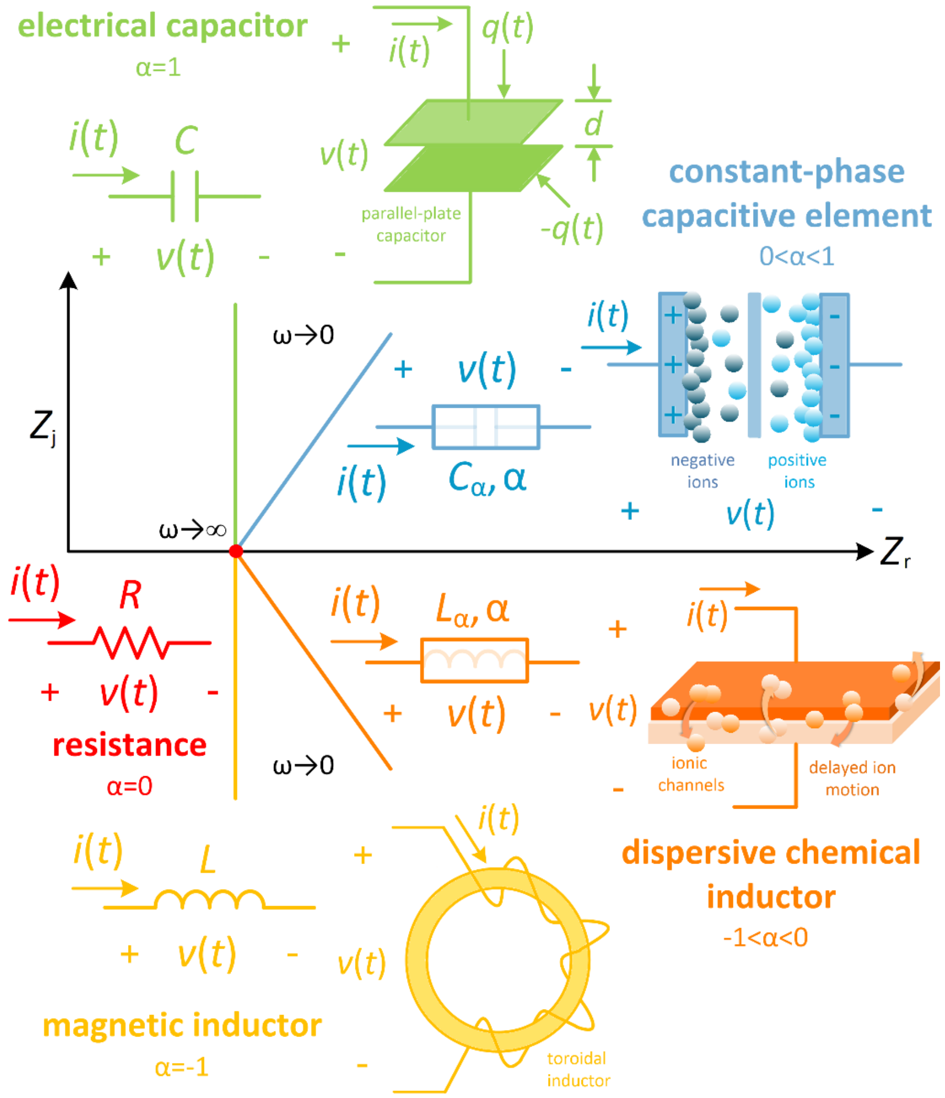
- **CPE impedance:** $Z(j\omega)=\dfrac{1}{Q(j\omega)^{\alpha}}$



CONSTANT PHASE ELEMENT (CPE)

Origin: Frequency-response

- **CPE impedance:** $Z(j\omega) = \dfrac{1}{Q(j\omega)^{\alpha}}$
  - Is the phase angle of the CPE impedance independent of the frequency?



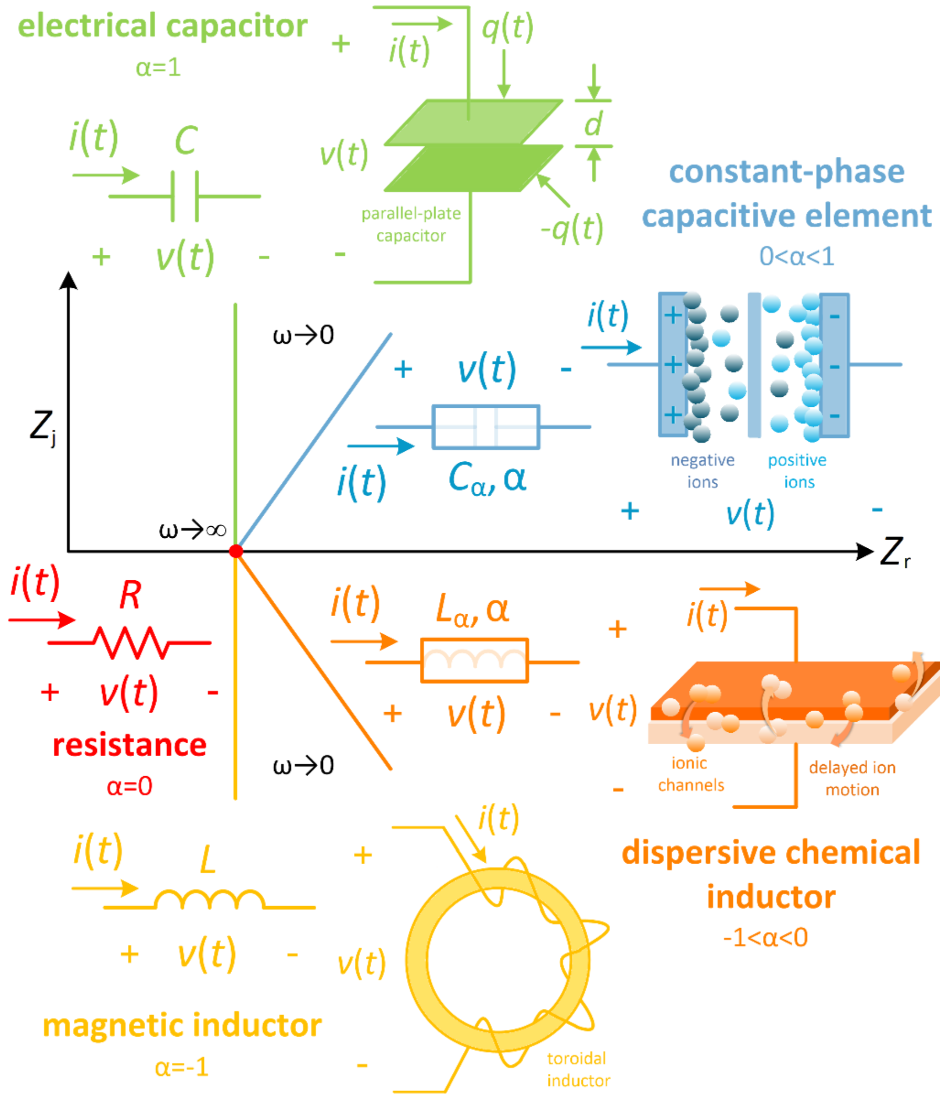
CONSTANT PHASE ELEMENT (CPE)

Origin: Frequency-response

- **CPE impedance:** $Z(j\omega)=\dfrac{1}{Q(j\omega)^{\alpha}}$
  - Is the phase angle of the CPE impedance independent of the frequency?

$$Z(j\omega)=\frac{1}{Q(j\omega)^{\alpha}}=\underbrace{\frac{1}{Q\omega^{\alpha}}}_{|Z(j\omega)|}\angle\underbrace{-\alpha\pi/2}_{\arg[Z(j\omega)]}$$

$$Z(j\omega)=\underbrace{\frac{1}{Q\omega^{\alpha}}\cos(\alpha\pi/2)}_{R(\omega)}-j\underbrace{\frac{1}{Q\omega^{\alpha}}\operatorname{sen}(\alpha\pi/2)}_{-X(\omega)}$$



CONSTANT PHASE ELEMENT (CPE)

Origin: Frequency-response

- **CPE impedance:** $Z(j\omega)=\dfrac{1}{Q(j\omega)^{\alpha}}$

  - Is the phase angle of the CPE impedance independent of the frequency?

$$Z(j\omega)=\frac{1}{Q(j\omega)^{\alpha}}=\underbrace{\frac{1}{Q\omega^{\alpha}}}_{|Z(j\omega)|}\angle\underbrace{-\alpha\pi/2}_{\arg[Z(j\omega)]}$$

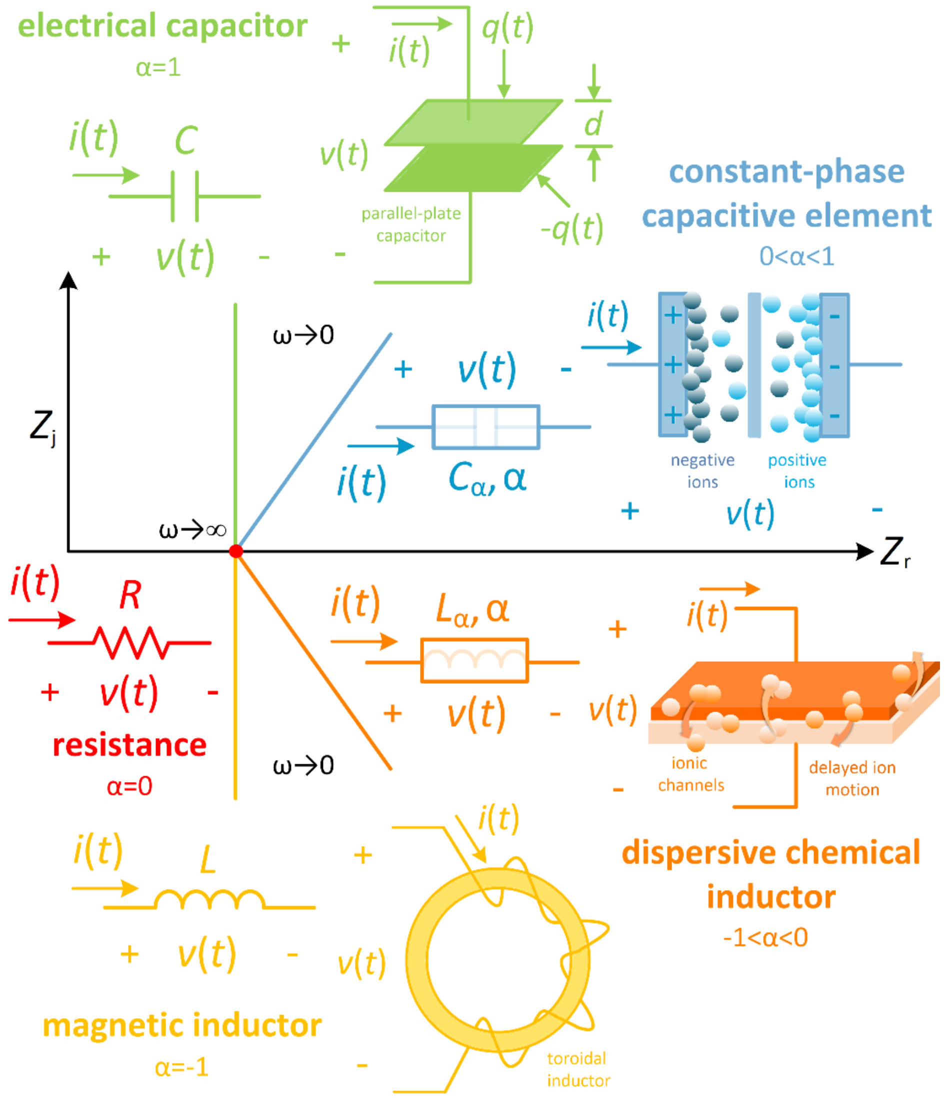
$$Z(j\omega)=\underbrace{\frac{1}{Q\omega^{\alpha}}\cos(\alpha\pi/2)}_{R(\omega)}-j\underbrace{\frac{1}{Q\omega^{\alpha}}\mathrm{sen}(\alpha\pi/2)}_{-X(\omega)}$$

- **Fractal impedance scaling in frequency:**

$$Z(jk\omega)=\frac{1}{Q(jk\omega)^{\alpha}}=k^{-\alpha}Z(j\omega)$$

- **Self-similarity:**

$$\frac{Z(j\omega)}{Z(jk\omega)}=k^{\alpha}$$



FRACTIONAL DYNAMICS OF R-CPE CIRCUITS

Transient response (0<α<1)

FRACTIONAL DYNAMICS OF R-CPE CIRCUITS

Transient response ($0<\alpha<1$)

- **Fractional ordinary differential equation, with constant coefficients:**

$$\frac{d^{\alpha}x(t)}{dt^{\alpha}}+\frac{x(t)}{\tau^{\alpha}}=0; \quad 0<\alpha<1$$

FRACTIONAL DYNAMICS OF R-CPE CIRCUITS

Transient response (0<α<1)

- **Fractional ordinary differential equation, with constant coefficients:**

$$\frac{d^{\alpha}x(t)}{dt^{\alpha}} + \frac{x(t)}{\tau^{\alpha}} = 0; \quad 0<\alpha<1$$

- **Time constant:**
  - (i) α exponent (Dimensional criteria).
  - (ii) Distributed relaxation times with a discernible average value experimentally measured: $\tau = (R_{Th}Q)^{1/\alpha}$

FRACTIONAL DYNAMICS OF R-CPE CIRCUITS

Transient response ($0<\alpha<1$)

- **Fractional ordinary differential equation, with constant coefficients:**

$$\frac{d^{\alpha}x(t)}{dt^{\alpha}}+\frac{x(t)}{\tau^{\alpha}}=0;\quad 0<\alpha<1$$

  - **Time constant:**

    (i) α exponent (Dimensional criteria).

    (ii) Distributed relaxation times with a discernible average value experimentally measured:

$$\tau=\left(R_{Th}Q\right)^{1/\alpha}$$

- **Mittag-Leffler function** arises naturally as the solution of fractional differential and integral equations;

FRACTIONAL DYNAMICS OF R-CPE CIRCUITS

Transient response ($0<\alpha<1$)

- **Fractional ordinary differential equation, with constant coefficients:**

$$\frac{d^{\alpha}x(t)}{dt^{\alpha}}+\frac{x(t)}{\tau^{\alpha}}=0; \quad 0<\alpha<1$$

  - **Time constant:**
    - (i) α exponent (Dimensional criteria).
    - (ii) Distributed relaxation times with a discernible average value experimentally measured:

$$\tau=\left(R_{Th}Q\right)^{1/\alpha}$$

- **Mittag-Leffler function** arises naturally as the solution of fractional differential and integral equations;

$$E_{\alpha}[-(t/\tau)^{\alpha}]=\sum_{k=0}^{\infty}\frac{[-(t/\tau)^{\alpha}]^{k}}{\Gamma(\alpha k+1)}$$

α = 1: Exponential function

FRACTIONAL DYNAMICS OF R-CPE CIRCUITS

Transient response ($0<\alpha<1$)

- **Fractional ordinary differential equation, with constant coefficients:**

$$\frac{d^{\alpha}x(t)}{dt^{\alpha}}+\frac{x(t)}{\tau^{\alpha}}=0; \quad 0<\alpha<1$$

  - **Time constant:**
    - (i) α exponent (Dimensional criteria).
    - (ii) Distributed relaxation times with a discernible average value experimentally measured:

$$\tau=\left(R_{Th}Q\right)^{1/\alpha}$$

- **Mittag-Leffler function** arises naturally as the solution of fractional differential and integral equations;

Characteristic time scales

$$E_{\alpha}[-(t/\tau)^{\alpha}]=\sum_{k=0}^{\infty}\frac{[-(t/\tau)^{\alpha}]^{k}}{\Gamma(\alpha k+1)}$$

$\alpha = 1$: Exponential function

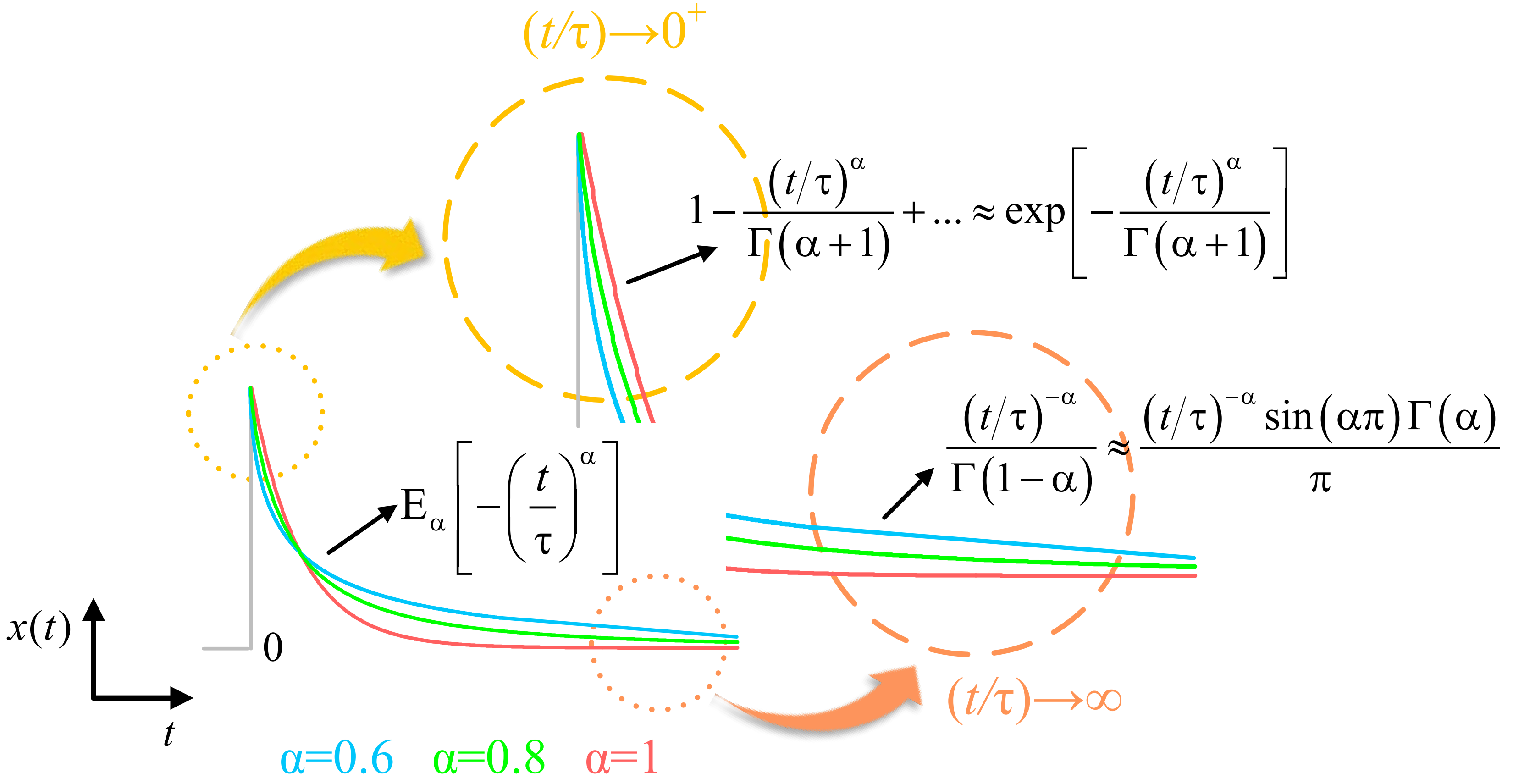
$$E_{\alpha}[-(t/\tau)^{\alpha}]\sim\begin{cases}1-\dfrac{(t/\tau)^{\alpha}}{\Gamma(\alpha+1)}+\ldots\sim\exp\left[-\dfrac{(t/\tau)^{\alpha}}{\Gamma(\alpha+1)}\right], & (t/\tau)\to 0^{+}\\ \dfrac{(t/\tau)^{-\alpha}}{\Gamma(1-\alpha)}\sim\dfrac{\mathrm{sen}(\alpha\pi)}{\pi}\left(\dfrac{\tau}{t}\right)^{\alpha}\Gamma(\alpha), & (t/\tau)\to\infty\end{cases}$$

FRACTIONAL DYNAMICS OF R-CPE CIRCUITS

Mittag-Leffler function vs. exponential behavior

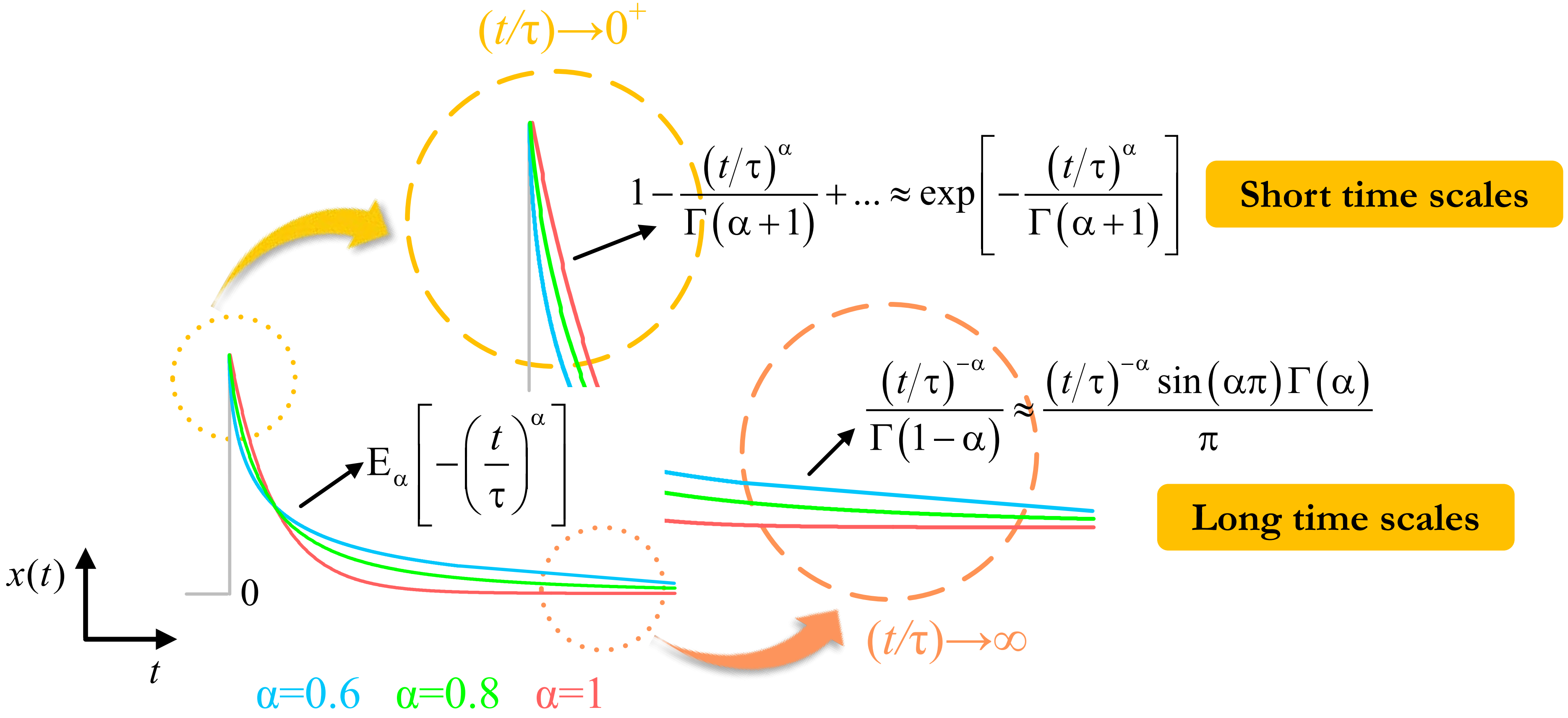
FRACTIONAL DYNAMICS OF R-CPE CIRCUITS

Mittag-Leffler function vs. exponential behavior



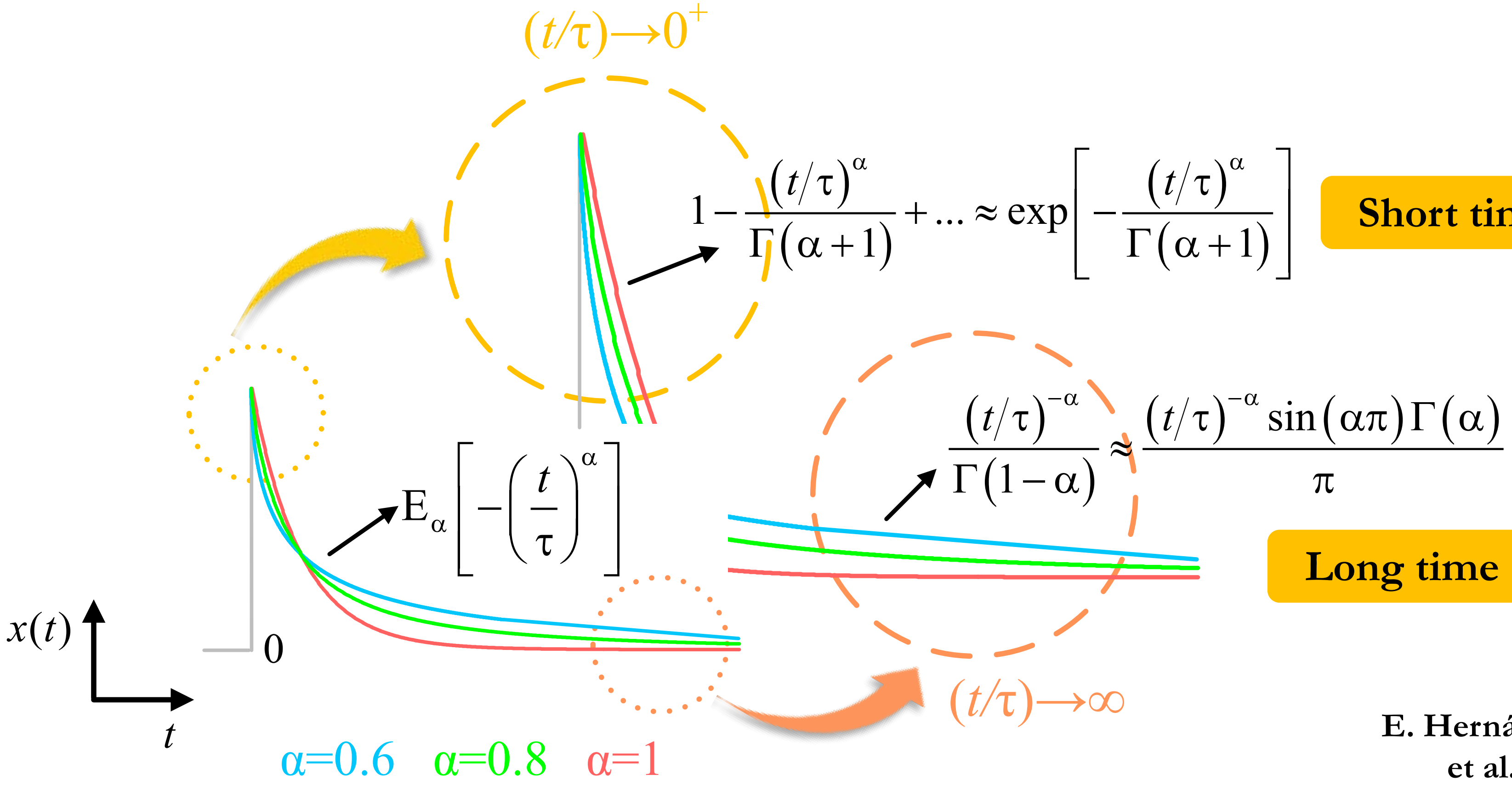
FRACTIONAL DYNAMICS OF R-CPE CIRCUITS

Mittag-Leffler function vs. exponential behavior

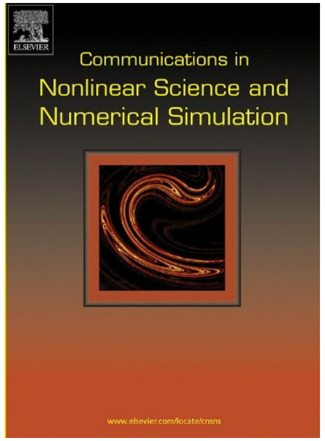


FRACTIONAL DYNAMICS OF R-CPE CIRCUITS

Mittag-Leffler function vs. exponential behavior



Mittag-Leffler function "draws" a transition from KWW ("stretched exponential", t^{α}) to an inverse power-law function, $t^{-\alpha}$.



E. Hernández-Balaguera et al. *Communications in Nonlinear Science and numerical simulations* 90 (2020) 105371

From basic electrical concepts to equivalent circuits that models physical phenomena

Enrique Hernández Balaguera
Department of Applied Mathematics, Materials Science and Engineering and Electronic Technology
Universidad Rey Juan Carlos, Madrid (Spain)

15-19 May 2023, Chania, Creta (Greece)
Erasmus+ Staff Training Mobility

Universidad Rey Juan Carlos

Hellenic Mediterranean University

Erasmus+

10th International Week
2nd ATHENA International Week